OPEN ACCESS INTERNATIONAL JOURNAL OF SCIENCE & ENGINEERING

SIMULATION OF FIR FILTER BANKS USING DAUBECHIES WAVELET WITH ORTHONORMAL BASIS FUNCTIONS

Kavya Dwivedi, Ravindra Pratap Narwari
Department of Electronic Engineering, MITS Gwalior,
kavyadwivedi23@gmail.com

Abstract: **State space analysis is an excellent method for design and analysis of control system. It can be applied to Non-linear, Time-invariant and MIMO systems. Proposed research work has presents a state space description for the minimal delay wavelet FIR filter banks by using special orthonormal basis function. This FIR structure guarantees BIBO stability, Robustness and provide Perfect reconstruction by verifying the Minimality, observability, and the minimal realization for the filter transfer function. Also proposed work has lesser delay compared to base research work.**

Keywords *— State space description, Wavelet, FIR filter bank, Orthonormal basis function, Paraunitary matrices, Reachable, Observable, Minimality, Eigen value.*

I INTRODUCTION

A Finite impulse response (FIR) Filter is a filter structure that can be used to operate with any kind of frequency response digitally. FIR is a filter whose impulse response is the finite period. In a FIR filter, there is no feedback. FIR filter is usually designed to be linear phase.

In digital Signal processing and Communication Wavelet transforms play a considerable role. Wavelet filters and Scaling play a significant role in the wavelet transform. They are used in the formulation of Discrete Wavelet Transform coefficient algorithm implemented by filter banks. Filter coefficients of FIR Filter are computed from Daubechies wavelet bases. Filter banks are constructed of low pass, bandpass and high pass filters used for the spectral decomposition and composition of signals. They play an important role in many signal processing applications, Scrambling image compression, adaptive signal processing and transmission of several signals through the same channel. The reason for their popularity is the fact that they easily allow the extraction of the spectral component of a signal while providing very efficient implementations. Many of the filter banks involve distinct sampling rates, they are also called as multi-rate systems.

The main idea of using filter banks is that in frequency domain its separate the signal under consideration into two or more signals or to fused two or more different signal into a single signal. A perfect reconstruction FIR filter should follow stability condition. Any space model that is both controllable and observable and has the same input output behaviour as the transfer function is said to be a minimal realization of the transfer function. Besides the minimality condition, reachability and observability conditions were achieved. This approach can also be valuable for use with nonlinear models.

P.P Vaidyanathan describes the matrices which are a Unitary matrix and Paraunitary matrix. These matrices are used in Signal processing, Robustness digital filtering, and multi-rate filter bank design. These matrices are employed in the finite field when the author needs to compute in the finite field. In this paper, the author has described various properties of these matrices. The author has also determined and well applied these matrices in group field (GF2).The Paraunitary matrix contains a degree one.[8]

P.P Vaidyanathan describes the parameter of perfect reconstruction (PR)FIR filter bank and this can be done with the help of orthogonal wavelet. For the PR of the FIR Quadrupture mirror filter(QMF) bank, the determinant of the polyphase matrix (E(z)) of analysis must be equal to delay. The author explores a different technique to describe such matrices. To solve problem author describe this in two problems that are first it describes the parameter of the lossless matrix and after that, it describes the parameter of the unimodular matrix. In this paper, the author uses different types of matrices for describing the

OPEN ACCESS INTERNATIONAL JOURNAL OF SCIENCE &ENGINEERING

parameter of the filter bank and also describes the problem in the parameter of the unimodular matrix that is converted into the problem in linear algebra. [7]

G Sherlock and D.M.Monro have proposed a technique for finding filter coefficient of Perfect reconstruction (PR) FIR filter of different length. The authors have described various parameters for determining filter coefficient as the author used Daubechies filter coefficient also they take different types of roots because parametrization is not unique, authors have also described the wavelets on the basis of a number of vanishing moment. In this work, authors have also described the symmetry properties of the space. [9]

Selvaraaju Murugesan & David B.H Tay have proposed a technique to preserve vanishing moment of the biorthogonal and orthogonal filter and also Rationalize the orthogonal and biorthogonal filter with perfect reconstruction. However, in the orthogonal filter, it shows energy preservation but does not show symmetry, due to this reason biorthogonal filter is preferred. For the regularity of the wavelet, Vanishing moment (VM) plays a significant role. Biorthogonal filter banks preserve many VMs at each stage but orthogonal filter bank preserves atmost one VMs. The authors have proposed another technique Zero dc leakage preservation which rationalizes the orthogonal filter. Complementary filters are used for the biorthogonal filter.[24]

Ramirez-Echeverria et al. proposed that for Minimal Square Error(MSE) in discrete time state space FIR filter. Authors have Estimated the optimal memory. There exist no relation between the optimal memory and filter order. In this estimation, we donot include any type of distortion and noise which is the significant property. Also optimal memory N_{OPT} can be determined with the help of mean square value(MSV). Authors have also applied learning cycle for determing N_{OPT}. [12]

II PROPOSED METHODOLOGY

In this work, we have represented the filter using State space analysis to formulate the output as a function of input filtered signal either for a low pass or high pass filter. The current input of the filter is a function of input state at a previous sample and also the input signal. In the state space for perfect reconstruction wavelet designing FIR filter banks with the help of special orthonormal basis functions. Orthonormal basis function play a significant role and its distinctive properties make them attractive in the modelling of dynamic systems. Orthonormal functions are the functions that are both orthogonal and normalized. Both these properties make them special and provide the best result. With the help of special orthonormal basis function in Figure. 1 we could find the coefficient of filter and filter coefficient is different for different wavelet.

Figure 1 shows the generation of orthonormal wavelets from angular parameters. These parameters will be used for calculating and determining the state space. The Procedure for descripting filter bank parameterization in state space is depicted in Figure.2. From Figure.2 we defined some parameter that describes the filter.

$u[k]$ is the input signal (which being filtered) at sample number k.

$x[k]$ the state space at sample k.

$x[k+1]$ the state space at sample $k+1$.

$y[k]$ the output (the filtered signal) at sample number k.

By following this procedure having the first delay operator is Z^{-1} and the remainders are Z^{-2}. $C_i = \sin(\theta_i)$ and $S_i = \cos(\theta_i)$, where $\theta = [\theta_1, \theta_2, \theta_3 \dots\dots\dots \theta_N]$ and $\theta_1 = \alpha_N$, $\theta_2 = \alpha_{N-1}$,..., $\theta_N = \alpha_1$. Also, the output $y1$ is the output signal for an input signal u which is filtered by the low-pass filter and $y2$ is the output to the input signal u which filtered by the high-pass filter. Therefore, the model in the state space is can be represented as follows.

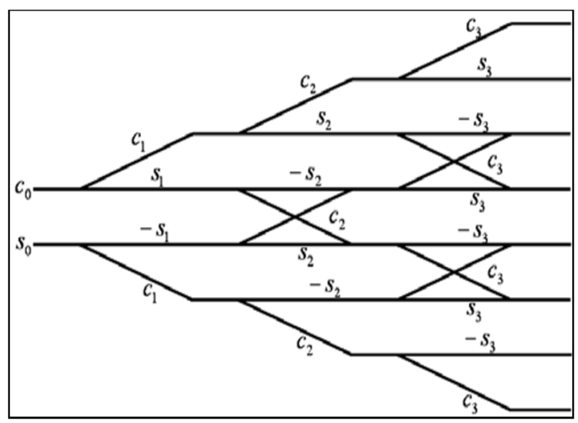

Figure 1: Generation of orthonormal wavelets from angular parameters

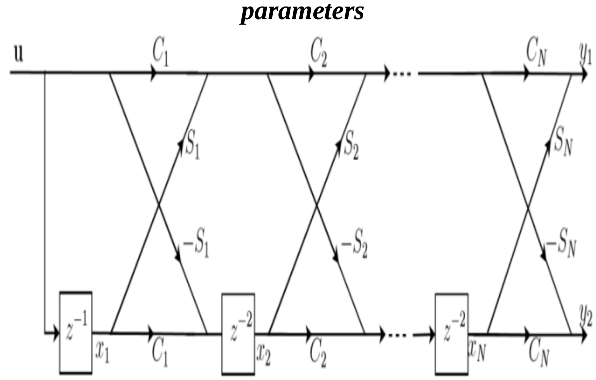

Figure 2: Procedure for filter bank parameterization in state space

The input and output signal at any sample K can be calculated from the previous values

$$x[k+1] = Ax[k] + Bu[k] \qquad \text{“(Eq.1)”}$$

$$y[k] = Cx[k] + Du[k] \qquad \text{“(Eq.2)”}$$

For input $x = [x_1, x_2, x_{3 \ldots\ldots} x_{2N-1}]^T$ and k denotes the k^{th} sampling instant. From figure.2 it follows equation (13)-(16) from [1]. With the help of this equation, we use to, find state matrix of the FIR filter.

The state matrix (or system matrix) A is given by

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 1 \\ C_1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ -S_2S_1 & C_2 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ -S_3C_2S_1 & -S_3S_2 & C_3 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \cdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ S_{N-1}S_1\prod_{i=2}^{N-2} C_i & S_{N-1}S_2\prod_{i=3}^{N-2} C_i & S_{N-1}S_3\prod_{i=4}^{N-2} C_i & \cdots & C_{N-1} & 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix},$$

and the input matrix B is given by

$$\mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ -S_1 \\ -S_2C_1 \\ \vdots \\ -S_{N-1}\prod_{i=1}^{N-2} C_i \end{bmatrix}.$$

The output matrix C and the feedthrough (or feed forward) matrix D presented and is given by

$$\mathbf{C} = \begin{bmatrix} S_1\prod_{i=2}^{N} C_i & S_2\prod_{i=3}^{N} C_i & \cdots & S_{N-2}\prod_{i=N-1}^{N} C_i & S_{N-1}C_N & S_N & 0 & \cdots & 0 \\ -S_NS_1\prod_{i=2}^{N-1} C_i & -S_NS_2\prod_{i=3}^{N-1} C_i & \cdots & -S_NS_{N-2}\prod_{i=N-1}^{N-1} C_i & -S_NS_{N-1} & C_N & 0 & \cdots & 0 \end{bmatrix}$$

Where

A= Coefficient of $x[k]$ follows the equation to get the root of the matrix element.

B= Coefficient of $u[k]$ for the $x[k+1]$ states.

C= Coefficient of $x[x]$ for output $y[k]$.

D= Coefficient of $u[k]$ for output $y[k]$.

$$\mathbf{D} = \begin{bmatrix} \prod_{i=1}^{N} C_i \\ -S_N\prod_{i=1}^{N-1} C_i \end{bmatrix},$$

Validation

The second step, after calculating the state space matrices A, B, C, and D. it should verify that they are the minimal realization of the transfer function H(Z) of the FIR Filter by verifying the observability and reachability of the obtained structure with n delays.

The observation condition is having the matrix $R_{C,A} = \begin{bmatrix} C \\ CA \\ CA^2 \\ . \\ . \\ CA^{n-1} \end{bmatrix}$ has the full rank n.

And the reachability condition is having the matrix $R_{A,B} = [B \;\; AB \;\; A^2B \;\; \ldots\ldots \;\; A^{n-1}B]$ has full rank n.

By coding the modelling equations using MATLAB 2013 as included in appendix A, the state space matrices can be obtained. A fully graphical user interface is created using MATLAB as shown in Figure.3, it enables the use easily to enter the angular parameters and after validating the input, its processes according to the model equation discussed above and generate the state space matrices A, B, C, and D and also calculate the coefficients of the filter. Furthermore, there is an option to validate the minimality of the realization and show an example of the outputs of the process.

III RESULT & DISCUSSION

The set of angular parameter for db2 is $\alpha = \{-1.3090, 2.0944\}$ is obtained with the help of parametrization of orthonormal wavelets. For db2, a $(2N-1) \times (2N-1)$ matrix is obtained for realization and $(4N-2) \times (2N-1)$ matrix is obtained for observability. So here N=2 for db2. The state matrices can be obtained from the formulation of A, B, C and D. The length of the wavelet filter that is filter coefficient is two times that number. For db2, length$= 2 \times 2 = 4$.

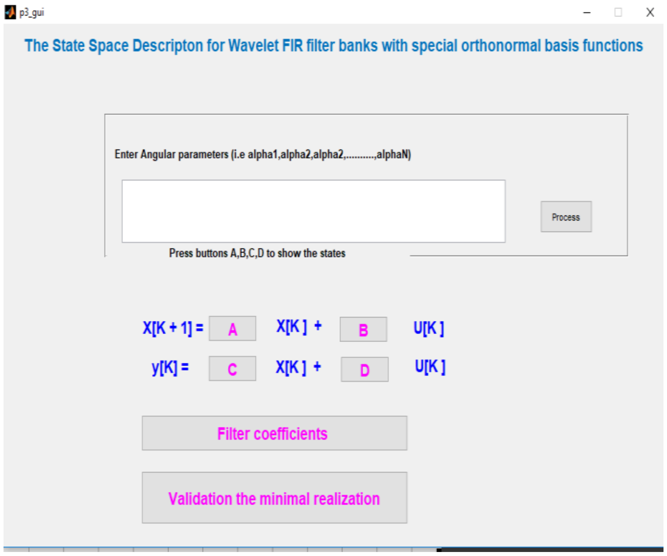

Figure 3 Graphical user interface for the operation

State Matrices

$$A=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ -0.5 & 0 & 0 \end{bmatrix},\ B=\begin{bmatrix} 1 \\ 0 \\ -0.866 \end{bmatrix}$$

$$C=\begin{bmatrix} 0.2241 & -0.9659 & 0 \\ 0.8365 & 0.2588 & 0 \end{bmatrix}$$

$$D=\begin{bmatrix} -0.1294 \\ -0.4830 \end{bmatrix}$$

Filter coefficient

$[0.1294, 0.2241, 0.8365, 0.4830]$

Validation

$$\boldsymbol{R_{C,A}}=\begin{bmatrix} 0.2241 & -0.9659 & 0 \\ 0.8365 & 0.2165 & 0 \\ 0 & 0 & -0.9659 \\ 0 & 0 & 0.2165 \\ 0.4830 & 0 & 0 \\ -0.1083 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{R_{A,B}}=\begin{bmatrix} 1 & 0 & 0.8660 \\ 0 & -0.8660 & -0.5000 \\ 0 & -0.500 & 0 \end{bmatrix}$$

The angular parameters for db3 are $=\{1.4653, 0.4998, -1.1797\}$. The number refers to the number of vanishing moments. Basically the higher the number of vanishing moments, the smoother the wavelet. In db3 the length is $2 \times 3 = 6$. So the length of filter coefficient are 6.

$$A=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0.3812 & 0 & 0 & 0 & 0 \\ 0.4441 & 0.8777 & 0 & 0 & 0 \end{bmatrix}$$

$$B=\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0.9245 \\ -0.1827 \end{bmatrix}$$

$$C=\begin{bmatrix} -0.0854 & 0.0505 & 0.9944 & 0 & 0 \\ 0.8069 & -0.4766 & 0.1053 & 0 & 0 \end{bmatrix}$$

$$D=\begin{bmatrix} 0.0352 \\ -0.3327 \end{bmatrix}$$

Filter coefficient

$[0.0352, -0.0854, -0.1350, 0.4598, 0.8069, 0.3327]$

Validation

$$\boldsymbol{R_{C,A}}=\begin{bmatrix} -0.0854 & 0.0505 & 0.9944 \\ 0.8069 & -0.4766 & 0.0968 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.4598 & 0.8728 & 0 \\ -0.1388 & 0.0850 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.3327 & 0 & 0 \\ 0.0324 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{R_{A,B}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.9245 & -0.1827 \\ 0 & 0.3812 & 0.4431 \\ 0 & 0 & 0.8114 \\ 0 & 0 & 0.3346 \end{bmatrix}$$

IV CONCLUSION

This work introduces a new state space description for wavelet finite impulse response filter by utilizing special orthonormal basis functions. The realization was optimized to be minimal and has the fewest delay for the specific transfer function of a FIR Filter. This realization characterizes the filter by its coefficients which are determined based on orthonormal basis functions. Furthermore, a complete graphical interface has been created to facilitate the process of entering the input parameters and it includes the validation of the observability and reachability of the obtained structure with n delay to be sure that the realization is minimal and there is no other structure has a lesser delay. This proposed work is also suitable for the nonlinear model. Also, the Eigen values of an are zero.

REFRENCES

[1] Julio C. Uzinski, Henrique M. Paiva, Marco A.Q. Duarte, Roberto K.H. Galvão, Francisco Villarreal, A state-space description for perfect-reconstruction wavelet FIR filter banks with special orthonormal basis functions, 2015

[2] L. Ljung, System Identification: Theory for the User, second ed., Prentice Hall, Upper Siddle River, 1999.

[3] P. Heuberger, P. Van Den Hof, B. Wahlberg, Modelling and Identification with Rational Orthogonal Basis Functions, Springer, London, 2005.

[4] J.B. Machado, Nonlinear systems modeling based on ladder-structured generalized orthonormal basis functions (Ph.D. thesis), Campinas, SP, Brazil, 2011 (in Portuguese).

[5] P.P. Vaidyanathan, Multirate Systems and Filter Banks, Prentice Hall, Upper Saddle River, NJ, 1993.

[6] J. Tuqan, P.P. Vaidyanathan, A state space approach to the design of globally optimal FIR energy compaction filters, IEEE Trans. Signal Process. 48(2000) 2822–2838.

[7] P.P. Vaidyanathan, How to capture all FIR perfect reconstruction QMF banks with unimodular matrices? IEEE Int. Symp. Circuits Syst. 1990 (3) (1990) 2030–2033.

[8] P.P. Vaidyanathan, Unitary and paraunitary systems in finite fields, IEEE Int. Symp. Circuits Syst. (1990) 1189–1192.

[9] B.G. Sherlock, D.M. Monro, On the space of orthonormal wavelets, IEEE Trans. Signal Process. 46 (1998) 1716–1720.

[10] C.K. Ahn, Strictly passive FIR filtering for state-space models with external disturbance, Int. J. Electron. Commun. 66 (2012) 944–948.

[11] P.S. Kim, An alternative FIR filter for state estimation in discrete-time systems, Digit. Signal Process. 20 (2010) 935–943.

[12] F. Ramirez-Echeverria, A. Sarr, Y.S. Shmaliy, Optimal memory for discrete-time FIR filters in state-space, IEEE Trans. Signal Process. 62 (2014) 557–561.

[13] Y.S. Shmaliy, Linear optimal FIR estimation of discrete time-invariant state-space models, IEEE Trans. Signal Process. 58 (2010) 3086–3096.

[14] Y.S. Shmaliy, L.J. Morales-Mendoza, FIR smoothing of discrete-time polynomial signals in state space, IEEE Trans. Signal Process. 58 (2010) 2544–2555.

[15] F. Ding, Y. Wang, J. Ding, Recursive least squares parameter identification algorithms for systems with colored noise using the filtering technique and the auxiliary model, Digit. Signal Process. 37 (2015) 100–108.

[16] Y. Liu, F. Ding, Y. Shi, An efficient hierarchical identification method for general dual-rate sampled-data systems, Automatica 50 (2014) 962–970.

[17] D. Wang, H. Liu, F. Ding, Highly efficient identification methods for dual-rate Hammerstein systems, IEEE Trans. Control Syst. Technol. (2015) in press http://dx.doi.org/10.1109/TCST.2014.2387216.

[18] D. Wang, F. Ding, L. Ximei, Least squares algorithm for an input nonlinear system with a dynamic subspace state space model, Nonlinear Dynam. 75(2014) 49–61.

[19] F. Ding, Combined state and least squares parameter estimation algorithms for dynamic systems, Appl. Math. Model. 38 (2014) 403–412.

[20] Y. Gu, F. Ding, J. Li, State filtering and parameter estimation for linear systems with d-step state-delay, IET Signal Process. 8 (2014) 639–646.

[21] F. Ding, State filtering and parameter estimation for state space systems with scarce measurements, Signal Process. 104 (2014) 369–380.

[22] F. Ding, L. Qiu, T. Chen, Reconstruction of continuous-time systems from their non-uniformly sampled discrete-time systems, Automatica 45 (2009)324–332.

[23] G. Strang, T. Nguyen, Wavelets and Filter Banks, Cambridge Press, Wellesley, MA, 1996.

[24] Selvaraaju Murugesan and David B. H. Tay, “New Techniques for Rationalizing Orthogonal and Biorthogonal Wavelet Filter Coefficients”, IEEE Transaction on circuit and system VOL. 59, NO. 3, MARCH 2012.

[25] Navneet Gupta & Ravindra Pratap Narwaria, “FIR filter design using artificial neural network” published in Research & Reviews: A Journal of Neuroscience (STM Journal) (2013), ISNN: 2277-6427 pp. 29-34.

[26] Navneet Gupta & Ravindra Pratap Narwaria, “Study of FIR filter designing by using Self Organizing Map neural network.” International Journal of advance Science and Technology (SERSC Journal) Vol. 70 (2014), ISNN: 2005-4238 pp. 1-10.

[27] Seema Verma , Ravindra Pratap Narwaria & Ravikant Prajapati, “Performance analysis of FIR digital filter using artificial neural network for ECG signal” published in Journal of Communication Engineering & systems, Volume 3, Issue 2, ISNN: 2249-8613, 2013, pp 28-32.

[28] Gaurav Jain & Ravindra Pratap Narwaria, “Artificial Neural Network Model for cutoff frequency calculation for denoising of ECG signal using FIR filter ”, published in International Journal of Signal Processing, Image Processing and Pattern Recognition (SERSC Journal) Vol. 10, No. 1 (2017), ISNN: 2005-4254 pp. 183-190.

[29] Vijay Barfa & Ravindra Pratap Narwaria, “Incremental Partial Update Strategies Over Distributed Networks”, published in Current Trends in Signal Processing (STM Journals), ISNN: 2277-6176, volume 5, issue 2, 2015, pp 1-8.

[30] Shruti B. Pawar, Dr. S. S. Pimplikar] Issues and Prospects of Public-Private Partnership Issue 1, Year: 2017, Pages:45-47, Volume 2 International Journal of Advance Scientific Research and Engineering Trends (IJASRET)ISSN 2456-0774

[31] Devashri Deshmukh, Ulhas B. Shinde, Shrinivas R. Zanwar Android Based Health Care Monitoring System Volume 2, ISSUE 7, Year: 2017, Pages:188-193 International Journal of Advance Scientific Research and Engineering Trends (IJASRET) ISSN 2456-0774

[32] Shivani Sharma & Ravindra Pratap Narwaria, “Noise Removal Techniques for ECG signal – A Survey Approach” published in Current Trends in Signal Processing (STM Journals), ISNN: 2277-6176, 2013, pp 19-23.

[33] Mitali Poojari, Dr. Geetha K. Jayaraj Runoff Modeling Using SWAT Hydro logical Model for Ghataprabha Basin Volume 2, Issue 1, Year: 2017, Pages:65-69,

[34] Shivani Sharma & Ravindra Pratap Narwaria, “Noise Reduction from ECG signal using Adaptive filter Algorithm” published in International Journal of Engineering Research and technology, Volume 3, Issue 7, ISNN: 2278-0181, July 2014, pp 437-440.

[35] Shivani Sharma & Ravindra Pratap Narwaria, “Performance Evaluation of various window techniques for Noise cancellation from ECG signal” published in International Journal of Computer Application, Volume 93 and No. 19, ISNN: 0975-8887, May 2014, pp 1-5.