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PARTIAL STANDARDIZATION IN TRACING MARGINAL CONTRIBUTION OF EXPLANATORY VARIABLES IN REGRESSION

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Abstract: Tracing relative importance of explanatory variables of regression model in application problem studies such as, social, agricultural, medical, engineering and industrial sciences attracts the researchers. It is essential to detect the importance of explanatory variables that contribute most to the response variable. In this paper an attempt is made to exemplify here by some empirical data to find out the importance of partial standardization in tracing marginal contribution of explanatory variables in regression.

Keywords: Multiple Regression, Partial regression, Relative importance, Standardized coefficient, Coefficient of determination

I INTRODUCTION

Partial standardization in tracing marginal contribution of explanatory variables in regression analysis, various measures are involved like R^2, Adj. R^2, C_p statistic, mean square error, *p* values and so on. Common modelings enable to compare regression coefficients with respect to size, but comparison is difficult when the variables are measured in different units. In this case one of the measures frequently used by some researchers is standardized regression coefficient. The work is carried out in the line of Bring (1994), Kruskal (1987), Afifi *et al*. (1990) and Goswami (2007). The main objections against standardized regression coefficients in tracing out relative importance are observed as:

1. Standardized regression coefficients are very difficult to interpret.
2. They are a mixture of two different concepts, viz., the estimated effect ($\hat{B}$) and standard deviation (S_j).
3. They are sample specific and unreliable to compare between two different samples of different features.

The standardized coefficients are dangerous to use while considering the magnitude of the coefficient as measure of relative importance because it is mostly affected by the range of the explanatory variables.

II STANDARDIZED REGRESSION COEFFICIENTS

Let the response variable y be related to *k* explanatory variables $x_1, x_2, \ldots, x_k$. Then we assume the model as given below:

$$y = B_0 + B_1 x_1 + B_2 x_2 + \ldots + B_k x_k + \mathcal{E}. \qquad (2.1)$$

The sample regression model corresponding to " Eq. 2.1" as

$$y_i = B_0 + B_1 x_{i1} + B_2 x_{i2} + \ldots + B_k x_{ik} + \mathcal{E}_i,$$
$$= B_0 + \sum_{j=1}^{k} B_j x_{ij} + \mathcal{E}_i, \; i=1, 2, \ldots, n.$$

Standardized regression coefficients are computed or can be calculated by using two popular scaling techniques, i.e. *unit normal scaling and unit length scaling,* both leading to the same result.

Using unit *normal scaling* for the explanatory and response variables we define new variabls,

$z_{ij} = (x_{ij} - \bar{x}_j)/s_j$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, k$ and

$y_i^* = (y_i - \bar{y})/s_y$, $i = 1, 2, \ldots, n$, where

$s_j^2 = \sum_{i=1}^{n}(x_{ij} - \bar{x}_j)^2/(n-1)$, is the sample variance of explanatory variable x_j and

$s_y^2 = \sum_{i=1}^{n}(y_i - \bar{y})^2/(n-1)$, is the sample variance of response variable.

Using these new variables, the regression model "Eq. 2.1" becomes

$y_i^* = \beta_1 z_{i1} + \beta_2 z_{i2} + \ldots + \beta_k z_{ik} + \varepsilon_i^*$, $i = 1, 2, \ldots, n$,

where $\beta_j = \hat{B}_j(S_j/S_y)$.

$$\text{i.e. } \begin{pmatrix} y_1^* \\ y_2^* \\ y_3^* \\ \vdots \\ y_n^* \end{pmatrix} = \begin{pmatrix} z_{11} & z_{12} & z_{13} & \cdots & z_{1k} \\ z_{21} & z_{22} & z_{23} & \cdots & z_{2k} \\ z_{31} & z_{32} & z_{33} & \cdots & z_{3k} \\ \vdots & \vdots & \vdots & & \vdots \\ z_{n1} & z_{n2} & z_{n3} & \cdots & z_{nk} \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \vdots \\ \beta_k \end{pmatrix} + \begin{pmatrix} \varepsilon_1^* \\ \varepsilon_2^* \\ \varepsilon_3^* \\ \vdots \\ \varepsilon_n^* \end{pmatrix}$$

i.e. $\boldsymbol{Y}^* = Z\boldsymbol{\beta} + \varepsilon^*$, say.

As centering the explanatory and response variables by subtracting $\bar{x}_j$ and $\bar{y}$ removes the intercept from the model. The least squares estimator of $\boldsymbol{\beta}$ is

$$\hat{\boldsymbol{\beta}} = (Z'Z)^{-1}Z'\boldsymbol{Y}^*, \qquad (2.2)$$

Using *unit length scaling*, we define again two new variables, $w_{ij} = (x_{ij} - \bar{x}_j)/\sqrt{S_{jj}}$, *i = 1, 2,..., n , j=1, 2,..., k* and $y_i^0 = (y_i - \bar{y})/\sqrt{SS_T}$, where

$S_{jj} = \sum_{i=1}^n (x_{ij} - \bar{x}_j)^2$ and $SS_T = \sum_{i=1}^n (y_i - \bar{y})^2$.

Using these new variables, w_{ij} which is having mean $\bar{w}_j = 0$ and length $\sqrt{\sum_{i=1}^n (w_{ij} - \bar{w}_j)^2} = 1$, the regression model "Eq. 2.1" becomes

$y_i^0 = \beta_1 w_{i1} + \beta_2 w_{i2} + \ldots + \beta_k w_{ik} + \varepsilon_i^{**}$,

i = 1,2,..., n, where $\beta_j = \hat{B}_j\sqrt{S_{jj}/SS_T}$

$$\text{i.e. } \begin{pmatrix} y_1^0 \\ y_2^0 \\ y_3^0 \\ \vdots \\ y_n^0 \end{pmatrix} = \begin{pmatrix} w_{11} & w_{12} & w_{13} & \cdots & w_{1k} \\ w_{21} & w_{22} & w_{23} & \cdots & w_{2k} \\ w_{31} & w_{32} & w_{33} & \cdots & w_{3k} \\ \vdots & \vdots & \vdots & & \vdots \\ w_{n1} & w_{n2} & w_{n3} & \cdots & w_{nk} \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \vdots \\ \beta_k \end{pmatrix} + \begin{pmatrix} \varepsilon_1^{**} \\ \varepsilon_2^{**} \\ \varepsilon_3^{**} \\ \vdots \\ \varepsilon_n^{**} \end{pmatrix}$$

i.e. $\boldsymbol{Y}^0 = W\boldsymbol{\beta} + \varepsilon^{**}$, say. The least squares regression coefficient is

$$\hat{\boldsymbol{\beta}} = (W'W)^{-1}W'\boldsymbol{Y}^0, \qquad (2.3)$$

It can be shown that if *unit normal scaling* is used, the $Z'Z$ matrix is closely related to $W'W$. i.e. $Z'Z$ = (n-1) $W'W$. So both scaling procedures produce the same set of dimensionless regression coefficients $\hat{\beta}$. In the *unit length scaling*, the matrix $W'W$ is in the form of a correlation matrix, that is

$$W'W = \begin{pmatrix} 1 & r_{12} & r_{13} & \cdots & r_{1k} \\ r_{12} & 1 & r_{23} & \cdots & r_{2k} \\ r_{13} & r_{23} & 1 & \cdots & r_{3k} \\ \vdots & \vdots & \vdots & & \vdots \\ r_{1k} & r_{2k} & r_{3k} & \cdots & 1 \end{pmatrix}$$

Where r_{ij} is the correlation coefficient of the i^{th} & the j^{th} components? The regression coefficients $\hat{\boldsymbol{\beta}}$ are usually called standardized regression coefficients. However for further use, the standardized regression coefficient given "Eq. 2.2" will be referred to *unit normal scaling* and that in "Eq. 2.3" to *unit length scaling*. As both the estimates of the standardized regression coefficients given in "Eq. 2.2" & "Eq. 2.3" give same set of dimensionless regression coefficients $\hat{\boldsymbol{\beta}}$, anyone can use any one of them. Using the standardized regression coefficient given "Eq. 2.2" the relationship between the original and standardized regression coefficients is

$$\hat{\beta}_j = \hat{B}_j(S_j/S_y), \ j = 1, 2,\ldots, k, \qquad (2.4)$$

Thus standardized coefficients are interpreted as standard deviation change in the response variable when the explanatory variables is changed by one standard deviation holding other variables constant. Walsh (1990) and Afifi and Clarke (1990) argued that (a) the variable with largest standardized regression coefficients $\hat{\beta}$ is most important and (b) the variable with the largest $\hat{\beta}$ contribute most to the prediction of *y*. It is clear from the arguments of Walsh (1990), Afifi and Clarke (1990) that eliminating the variable with the largest $\hat{\beta}$ from the equation would cause the largest reduction in R^2 . But their arguments seem to be sample specific. It will be clear from the following two numerical examples.

Here below some of the definitions are given for future uses.

A. Coefficient of Determination:

Coefficient of determination is defined as the ratio of explained sum of squares to the total sum of squares and is denoted by R^2.

B. Adjusted or corrected R^2:

Adjusted or corrected R^2 is given by

Adj. R^2 = 1- $[1 - R^2]\left[\frac{n-1}{n-p}\right]$, where *n* is the number of observation and *p* is the number of parameters in the equation. R^2 is the coefficient of determination having *p* parameters.

Numerical Example 2.1.

Data of Patchouli production as response variables based on five explanatory variables i.e.

1) Doses of chemical fertilizer like NPK,

2) Doses of organic manure like cow dung & vermin compose,

3) Application of micronutrients like mg, Zn…etc.,

4) Potentiality of irrigation,

5) Proper weed management i.e. manual + chemical, are collected from M/S B.V. Aromatics Oil Industry, Kaliabor, Nagaon (Assam), during (2003 A.D.- 2015 A.D.) the 12 years observations. Unit area data are 1hectare of land, i.e.7.5 Bighas. On an average, 1 Quintal of dry Patchouli leafs produces 2.5 kilograms of Patchouli oil.

y: P_ PRODUCTION (patchouli oil production) in kg.

x_1:C_FERTILIZER (doses of chemical fertilizer like, NPK) in kg.
x_2: O_MANURE (doses of organic manure like, cow dung & vermin compose) kg.
x_3: M_NUTRIEN (application of micronutrients like, mg, Zn….etc.) in kg.
x_4:IRRIGATION (potentiality of irrigation) in no.
x_5:W_MANAGEMENT (proper weed management i.e. manual + chemical) in no.

For a regression y on x_1,x_2, x_3, x_4 and x_5 we have the following correlation matrix.

$$\begin{array}{c} \\ y \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} \begin{array}{c} \begin{array}{cccccc} y & x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{pmatrix} 1 & 0.631 & 0.532 & 0.381 & 0.648 & 0.728 \\ \cdots & 1 & 0.150 & 0.014 & 0.532 & 0.600 \\ \cdots & \cdots & 1 & 0.701 & 0.254 & 0.378 \\ \cdots & \cdots & \cdots & 1 & 0.201 & 0.124 \\ \cdots & \cdots & \cdots & \cdots & 1 & 0.681 \\ \cdots & \cdots & \cdots & \cdots & \cdots & 1 \end{pmatrix} \end{array}.$$

Table 1 Standardized Regression Coefficient and their respective t – values

Standardized regression coefficients	$\hat{\beta}_1$ = 0.309	$\hat{\beta}_2$= 0.211	$\hat{\beta}_3$= 0.153	$\hat{\beta}_4$ = 0.181	$\hat{\beta}_5$= 0.321
t – values	01.101	0.621	0.474	0.578	0.914

R^2 =0.716 and Adj.R^2 =0.479.

The variable x_5 has the largest standardized coefficient. But it may not imply that the largest reduction in R^2 would be caused by eliminating the largest standardized coefficient variable x_5 .

$R^2_{y.x_2,x_3,x_4,x_5}$ = 0.658 and Adj.$R^2_{y.x_2,x_3,x_4,x_5}$= 0.463.

$R^2_{y.x_1,x_3,x_4,x_5}$ = 0.697 and Adj.$R^2_{y.x_1,x_3,x_4,x_5}$ = 0.525.

$R^2_{y.x_1,x_2,x_4,x_5}$ = 0.705 and Adj.$R^2_{y.x_1,x_2,x_4,x_5}$ = 0.536.

$R^2_{y.x_1,x_2,x_3,x_5}$ = 0.700 and Adj.$R^2_{y.x_1,x_2,x_3,x_5}$ = 0.528.

$R^2_{y.x_1,x_2,x_3,x_4}$ = 0.676 and Adj.$R^2_{y.x_1,x_2,x_3,x_4}$ = 0.491.

Table 2 Variables eliminated, Reduction in R^2, and Reduction in Adj.R^2

Variables eliminated	Reduction in R^2	Reduction in Adj.R^2
x_1	0.058	0.016
x_2	0.019	-0.046
x_3	0.011	-0.057
x_4	0.016	-0.049
x_5	0.040	-0.012

Observing the R^2 values from Table 2, it is clear that largest reduction in R^2 occurs when x_1 is removed, not when the variable with the largest standardized coefficient i.e. x_5 is removed. Similarly, same thing happens while using Adj.R^2.

Numerical Example 2.2.

For regression of y on x_1, x_2 and x_3 we have the following correlation matrix.

$$\begin{array}{c} \\ y \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cccc} y & x_1 & x_2 & x_3 \end{array} \\ \begin{pmatrix} 1 & 0.661 & -0.258 & 0.473 \\ \cdots & 1 & -0.106 & 0.451 \\ \cdots & \cdots & 1 & -0.192 \\ \cdots & \cdots & \cdots & 1 \end{pmatrix} \end{array}$$

The above data has been collected from "The Assam Co-operative Jute Mills Ltd, Silghat," Nagaon (Assam) where variables,

y: yearly production of sacking multiple fiber yarn, hessian laminated cloth bags in metric tons.
x_1: capital employed (share capital fund + reserve and surplus fund) rupees in lakh.
x_2: no. of skill labour.
x_3: jute procured in metric tons.

The Standardized regression coefficients are,

$\hat{\beta}_1$ = 0.558, $\hat{\beta}_2$= -0.162, $\hat{\beta}_3$= 0.191.

R^2 = 0.500 and Adj.R^2 = 0.364.

The variable x_1 has the largest standardized coefficient.

$R^2_{y.x_2,x_3}$= 0.253 and Adj.$R^2_{y.x_2,x_3}$= 0.129.

$R^2_{y.x_1,x_3}$= 0.475 and Adj.$R^2_{y.x_1,x_3}$= 0.388.

$R^2_{y.x_1,x_2}$= 0.472 and Adj.$R^2_{y.x_1,x_2}$=0.384.

Table 3 Variables eliminated, Reduction in R^2, and Reduction in Adj.R^2

Variables eliminated	Reduction in R^2	Reduction in Adj.R^2
x_1	0.247	0.235
x_2	0.025	-0.024
x_3	0.028	-0.020

Observing the R^2 values from Table 3, it is clear that the largest reduction in R^2occurs when x_1, the variable with the largest standardized regression coefficient, is removed. Similar observation is made based on Adj.R^2.

Both the Numerical Examples 2.1 & 2.2 reveal that "*Reduction in R^2 caused by eliminating the variable with largest coefficient is sample specific*."

III INCONSISTENCY IN THE CALCULATION OF STANDARDIZED COEFFICIENT

Consider the above Numerical Example 2.1.

y (P_ PRODUCTION) = B_0+ B_1(C_FERTILIZER) + B_2(O_MANURE) + B_3 (M_NUTRIENT) +B_4 (IRRIGATION) +B_5 (W_MANAGEMENT) + Ɛ , (3.1)

here, B_1represents the expected change in the response y (P_PRODUCTION), per unit change in (C_FERTILIZER) when all the remaining explanatory variables viz., (O_MANURE), (M_NUTRIENT), (IRRIGATION), (W_MANAGEMENT) are constant.

The estimated standardized regression coefficients for unit normal scaling are as follows.

$$\hat{\beta}_j = \hat{B}_j\ (s_j/s_y)\ ,\quad j = 1, 2, 3, 4 \text{ and } 5, \qquad (3.2)$$

For a particular j = 1, $\hat{\beta}_1 = \hat{B}_1\ (s_1/s_y)$,

where, $s_1$is the standard deviation of x_1 i.e. C_FERTILIZER and s_y is the standard deviation of y i.e. P_ PRODUCTION in the sample.

The inconsistency of $\hat{B}_1$lies in the fact that $\hat{B}_1$ and s_1 refer to different populations. The regression coefficient, $\hat{B}_1$, is interpretable only under the restriction that x_2 i.e.O_MANURE, x_3 i.e.M_NUTRIENT, x_4 i.e. IRRIGATION, x_5 i.e. W_MANAGEMENT are constant where as s_1 measures the spread of C_FERTILIZER over all the sample regardless of other variables. Similar problems are found for $\hat{B}_1$, $\hat{B}_2$, $\hat{B}_3$ and $\hat{B}_5$. To overcome this problem Bring (1994) suggested a new approach, known as partial standardization. He gave a simpler definition of partial standardized coefficient $\hat{\beta}_j^*$as a product of regression coefficient $\hat{B}_j$ and partial standard deviation s_j^*. i.e.

$$\hat{\beta}_j^* = \hat{B}_j\ .\ s_j^*, \qquad (3.3)$$

where $s_j^* = \frac{s_j}{\sqrt{VIF_j}}\sqrt{\frac{n-1}{n-k}}$, $VIF_j = \frac{1}{1-R_j^2}$, n = number of observations and k = number of explanatory variables and R_j^2 is coefficient of determination when x_j is regressed on k-1 other explanatory variables. Partial standardized coefficient $\hat{\beta}_j^*$ defined in relation "Eq. 3.3" omits s_y that is incorporated in standardized regression coefficient i.e. relation "Eq. 3.2". As all the coefficients having s_y as a common denominator, so omission of s_y does not change the relationship between the partial standardized coefficients. From the data of Numerical Example 2.1, here below we present Table 4, incorporate the standardized coefficients $\hat{\beta}_j$ and partial standardized coefficients $\hat{\beta}_j^*$ with corresponding explanatory variables.

Table 4 Standardized Coefficient and their respective Partial Standardized Coefficient

Variable	Standardized coefficients $\hat{\beta}_j$	Partial standardized coefficients $\hat{\beta}_j^*$
x_1	0.309	0.300
x_2	0.211	0.196
x_3	0.153	0.129
x_4	0.181	0.157
x_5	0.321	0.249

The largest value of standardized coefficients $\hat{\beta}_j$ is 0.321, corresponding to the explanatory variable x_5. However, in the Table 2, the largest reduction in R^2 occurs when x_1 is eliminated. So we cannot decide which variable is to change. Now referring to the partial standardized coefficients $\hat{\beta}_j^*$, we can see that the largest value of $\hat{\beta}_j^*$ is 0.300 corresponding to the explanatory variable x_1, so the reduction in R^2 is directly related to partial standardized coefficients.

IV RELATIONSHIP BETWEEN PARTIAL STANDARDIZED COEFFICIENTS AND REDUCTION IN R^2

For selecting explanatory variables which is closely related with response variable, t-values of each explanatory variable is commonly used based on the test of significance. It is generally known that,

$t_j = \frac{\hat{B}_j}{s_{\hat{B}_j}}$, where $s_{\hat{B}_j} = \frac{s_e}{\sqrt{\sum(x_{ij}-\bar{x}_j)^2(1-R^2{}_j)}}$, j = 1, 2, ..., k,

and s_e is the standard deviation of the residuals.R_j^2 is the coefficient of determination obtained from regressing x_j on k-1 other explanatory variables.

$$\frac{t_j}{t_{j+1}} = \frac{\hat{B}_j}{\hat{B}_{j+1}} \times \frac{\sqrt{\sum(x_{ij}-\bar{x}_j)^2}\sqrt{(1-R_j^2)}}{\sqrt{\sum(x_{ij+1}-\bar{x}_{j+1})^2}} \times \frac{1}{\sqrt{(1-R_{j+1}^2)}}.$$

Considering Eq. (3.3), we have

$$\frac{\hat{\beta}_j^*}{\hat{\beta}_{j+1}^*} = \frac{\hat{B}_j}{\hat{B}_{j+1}} \times \frac{\left(\sqrt{\sum(x_{ij}-\bar{x}_j)^2}/\sqrt{n-k}\right)}{\left(\sqrt{\sum(x_{ij+1}-\bar{x}_{j+1})^2}/\sqrt{n-k}\right)} \times \frac{\sqrt{(1-R_j^2)}}{\sqrt{(1-R_{j+1}^2)}},$$

for which , it is seen that

$$\frac{\hat{\beta}_j^*}{\hat{\beta}_{j+1}^*} \cong \frac{t_j}{t_{j+1}} \qquad (3.4)$$

Hence instead of comparing partial standardized coefficients, we could compare t - values, because t - values are directly related to R^2 values. The squared t - value is

$$t_j^2 = \frac{(R^2 - R^2_{y.1,2,3,\ldots,j-1,j+1,\ldots k})}{(1-R^2)/(n-k-1)} \quad (3.5)$$

Table 5 Comparing t-values and their respective Partial Standardized Coefficient

$t_1/t_2 = 1.7 \cong \hat{\beta}_1^*/\hat{\beta}_2^* = 1.67$
$t_2/t_3 = 1.47 \cong \hat{\beta}_2^*/\hat{\beta}_3^* = 1.5$
$t_3/t_4 = .82 \cong \hat{\beta}_3^*/\hat{\beta}_4^* = .82$
$t_4/t_5 = .63 \cong \hat{\beta}_4^*/\hat{\beta}_5^* = .63$

t - value of an explanatory variable is related to increment in R^2 and this increment related to how much it is possible to increase R^2. From the "Eq. 3.4" it is clear that comparing t - value is equivalent to considering the reduction in R^2, obtained by eliminating each of the variables. An evidence from example based on Patchouli production with the largest t - value, i.e. t = 1.101 against variable x_1, associated with highest partial standardized regression coefficient i.e. $\hat{\beta}_j^* = 0.300$. Similar inference can be made for the Numerical Example 2.2.

The new standardized regression coefficient is explored to answer three basic questions associated with relative importance. The questions are,

(a) What is the effect of changing a variable given other variables held constant?

(b) How much is it possible to change variable without changing the other explanatory variable?

(c) Why is partial standard deviation preferred to ordinary standard deviation?

First question is naturally answered by regression coefficient $\hat{B}_j$, because the usual interpretation of regression coefficients is the expected change in y when x_i is changed by one unit when other variables are constant.

In regards to the second question the study reveals that as the largest partial standardized coefficient associates with the largest t - value and is the product of s_j^* and $\hat{B}_j$, it gives an indication of which variable to change.

Third one is testified by the Numerical Examples 2.1. and 2.2. The Table 2 and Table 4 of the Numerical Example 2.1. reflect that the reduction in R^2 is directly related to partial standardized coefficients. From the Table *5*, shows that comparing partial standardized coefficients equivalent to comparing t- values.

V CONCLUSION

Tracing relative importance of explanatory variables in regression model attracts the researchers those involved especially in applied statistics. Several measures are used to compare the relative importance of the explanatory variables like t- values, regression coefficients, standardized regression coefficients, contribution to R^2 etc. In this paper some illustrative examples are given to explain the use of standardized regression coefficients and the problem of how to standardize them. The study reveals that it is not recommended to use of ordinary standard deviations in all situations to standardized regression coefficients however, partial standard deviations should be preferred and while we attempt to use any measures there is need for understanding the real situation.

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