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AN EXTENSION OF ORTHOGONAL ARRAY

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Abstract: In this paper, starting from an existing orthogonal array, a new orthogonal array is constructed by increasing the number of rows and also the number of columns.

Keywords: Orthogonal array, q-plet

I INTRODUCTION

The concept and notion of orthogonal arrays are due to Rao [8]. In his literature, imposing the concept of orthogonal arrays, many constructions of factorial designs are proposed and also proposed some relations among the parameters of the orthogonal array. Those had been improved by Bush [2] that for an orthogonal array (s^t, k, s, t) of index unity, if $t \le s$, then $k \le s+t-1$ when s is even and $k \le s+t-2$ when s is odd and $t \ge 3$. Further, he proposed as an improvement that in an orthogonal array (s^t, k, s, t) of index unity, if $s \le t$, then $k \le t+1$.

In the literature of Bose and Bush [1] it had been proved that for orthogonal array (λs^2, k, s, 3), $k \le (\lambda s^2-1)/(s-1)+1$. And if $\lambda-1$ is not divisible by s-1, then for the orthogonal array (λs^2, k, s, 3), $k \le (\lambda s^2-1)/(s-1) - [\sqrt{\{1+4s(s-1-b)\}}-(2s-2b-1)]/2$, b being the remainder on dividing λ-1by s-1.

A three symbol Partially array of strength (2m+1) has been constructed in the literature of Sharma [9], using image method of Dey *et al.* [3] on a tactical configuration (α-β-k-v) converted into design parameters by standard relationship. Gupta [5] has reviewed the method(s) of constructing orthogonal arrays (both symmetric and mixed) by exploiting the concept of resolvable (symmetric) orthogonal arrays and resolvable mixed orthogonal arrays. Several series of symmetric and mixed orthogonal arrays obtained in the literature by using the notion of Kronecker product and Kronecker sum of orthogonal arrays have also, been described. Further, he has proposed some general methods of obtaining orthogonal arrays and mixed orthogonal arrays. Dey and Midha [4] and Sinha, *et al.* [12] have contributed many series of orthogonal arrays. Hedayat *et al.* [6] have contribution to the orthogonal array in connection with its application and existence.

For the orthogonal array (s^2[(s-1)t+1], k, s, 2) the maximum number of constraints equals $s^2t+s+1=k^*$, say. Then the orthogonal array (s^2[(s-1)t+1], k^*, s, 2) will be called the maximal array and the orthogonal array (s^2[(s-1)t+1], k, s, 2) is said to have a deficiency $d=k^*-k$. Shrikhande and Bhagwandas [10] proved that an orthogonal array (s^2[(s-1)t+1], k, s, 2) can be embedded into a maximal array if (a) the deficiency is 1 for any value of s or (b) the deficiency is 2 for s=2 or 3. When s=2, the result reduces to the embedding problem of Hadamard matrices, Shrikhande and Bhagwandas [11].

II PRELIMINARIES

In this section some definitions, notations and proposition are laid down for the future use.

A. Balanced Array:

A k×N- matrix B with intries from a set S containing s symbols($s \ge 2$) is said to be a balanced array with s symbols, k constraints, N assemblies and strength t, if every t×N-submatrix of B contains the ordered t×1 column vector $(x_1, x_2, \ldots, x_t)^T$, $x_i \epsilon S$, $\lambda(x_1, x_2, \ldots, x_t)$ times where $\lambda(x_1, x_2, \ldots, x_t)$ is a non-negative integer and is invariant under any permutation of $x_1, x_2, \ldots, x_t$.

If $\lambda(x_1, x_2, \ldots, x_t)$ has a constant value, λ, say for all $x_1, x_2, \ldots, x_t$, the matrix B is called an orthogonal array due to Rao [7], which will be denoted by OA(N, k, s, t) and λ is called the index of the orthogonal array. Obviously, $N=\lambda s^t$.

Proposition 2.1:

An OA(N, k, s, t) is also again OA(N, k, s, t-1).

Proof. Let 0, 1, …, s-1 are the s symbols of the OA(N, k, s, t) with the index λ. Consider a t-plet $(\theta_1, \theta_2, ..., \theta_i, \ldots, \theta_t)^T$ where θ's ϵ {0, 1, …, s-1}. In any t×N-submatrix A_1 of OA(N, k, s, t), the t-plet occurs λ times where λ is given by $N=\lambda s^t$. As θ_i can take any one of the elements of {0, 1, …, s-1}, all the possible t-plets $(\theta_1, \theta_2, ..., \theta_{i-1}, \emptyset_i, \theta_{i+1} \ldots, \theta_t)^T$ where $\emptyset_i$ is a particular value of θ_i, occurs equal number of times λ. As θ_i may take any one of the s elements of the set {0, 1, …, s-1}, the (t-1)-plet $(\theta_1, \theta_2, ..., \theta_{i-1}, \theta_{i+1} \ldots, \theta_t)^T$ occurs $s\lambda$ times among all the possible N columns vector in the (t-1)×N-submatrix of A_1 (formed by neglecting the ith row of A_1).Thus, an OA(N, k, s, t) with the index λ is also again OA(N, k, s, t-1) with the index $s\lambda$.

III CONSTRUCTION

In this section, we propose the construction of a new orthogonal array, starting from an existing orthogonal array.

Theorem 3.1:

An OA(N^*, k^*, s^*, t^*) can be extended to another OA($N=s^*N^*$, $k=k^*+1$, $s= s^*$, $t=t^*$).

Proof. Let A^* be the OA(N^*, k^*, s^*, t^*) with its symbols 0, 1, …, s^*-1. Obviously, its index $\lambda^*_{t^*}$ is given by $N^*= \lambda^*_{t^*}(s^*)^{t^*}$. Define a new configuration, A, as given by

$$A= \begin{bmatrix} 0J & 1J & 2J & \dots & (s*-1)J \\ A^* & A^* & A^* & \dots & A^* \end{bmatrix},$$

$= \begin{bmatrix} B \\ A^{**} \end{bmatrix}$, say, where J is a $1\times N^*$-matrix with identity element "1" only.

Then all the symbols in A are 0, 1, …, s^*-1. Thus $s=s^*$. Clearly, it is seen that A is of order $(k^*+1)\times s^*N^*$ as A^* is of order order $k^*\times N^*$. Obviously, $N= s^*N^*$, $k=k^*+1$.For finding the index of the required orthogonal array A, it will be done into two parts:- (i) without concerning B, (i) with concerning B.

(i) Without concerning B: In A^* any q-plet ($q\leq t^*$) of symbols 0, 1, …, s^*-1 gets replicated equal number of times, λ_q, say. So, in A^{**} , the same q-plet gets replicated $s^*\ \lambda_q$ times.

(ii) With concerning B: Take a q-plet, $(\alpha, \beta_1, \dots, \beta_{q-1})^T$ from all the possible different $(s^*)^q$ q-plets. From the Proposition 2.1 we know that $N^*= \lambda^*_{q-1}(s^*)^{q-1} = \lambda^*_q(s^*)^q$. That is, $\lambda^*_{q-1}= s^*\ \lambda^*_q$. Any (q-1)-plet replicates λ_{q-1} times in A^*. So, the same (q-1)-plet replicates $s^*\ \lambda^*_{q-1}$ in A^{**}. Consider any q rows from A including the first row (i.e. B) i.e. any $q\times N$-submatrix including B. From the above, it is learnt that the (q-1)-plet $(\beta_1, \dots, \beta_{q-1})^T$ replicates λ^*_{q-1} (i.e. $s^*\ \lambda^*_q$) times among all the possible column vectors i.e. $(q-1)\times 1$-submatrix in A^{**}. As any one of the symbols 0, 1, …, s^*-1 appears with all the columns of A^* in A. The q-plet $(\alpha, \beta_1, \dots, \beta_{q-1})^T$ gets replicated λ^*_{q-1} i.e. $s^*\ \lambda^*_q$ times in the $q\times N$-submatrix of A (including B). Hence the index of the required orthogonal array is $s^*\ \lambda^*_q$.

Applying the Theorem 3.1 p times gives a corollary as given below.

Corollary 3.1:

An OA(N^*, k^*, s^*, t^*) can be extended to another OA($N=(s^*)^pN^*$, $k=k^*+p$, $s= s^*$, $t=t^*$) with index $(s^*)^p\lambda^*_q$.

An example of the Theorem 3.1 follows.

Example 3.1:

Take the OA(8, 3, 2, 2) given by

$$A^*= \begin{bmatrix} 0&0&0&0&1&1&1&1 \\ 0&0&1&1&0&0&1&1 \\ 0&1&0&1&0&1&0&1 \end{bmatrix}.$$

Then, using the Theorem 3.1 the resultant OA(16, 4, 2, 2) is given by

$$A= \begin{bmatrix} 0&0&0&0&0&0&0&0&1&1&1&1&1&1&1&1 \\ 0&0&0&0&1&1&1&1&0&0&0&0&1&1&1&1 \\ 0&0&1&1&0&0&1&1&0&0&1&1&0&0&1&1 \\ 0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1 \end{bmatrix}.$$

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