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A RESTRICTION ON t-DESIGNS FOR AFFINE RESOLVABITY

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Abstract: *This paper proposes that for affine resolvability of t-design, the number of treatments is strictly less than number of blocks.*

Keywords: *t-design, Affine resolvable block design.*

I INTRODUCTION

The important of t-designs arises due to their statistical optimality, desirable symmetries for analysis and interpretations. It is also used for constructing other important designs and structures such as Youden designs, generalized Youden designs, optimal fractional factorial designs, balanced arrays, combinatorial fitting systems. Kageyama (1973) had presented a construction method of a 3-design. A 3-design is called a doubly BIB design by Calvin (1954), who also gave a statistical analysis for an experiment based on 3-designs and the utility of 3-design. Kirkman (1847) gave a construction of $S(3, 4, 2^n)$ and showed that an $S(2, 3, v)$ exist iff v=1 or 3(mod6), where $S(t, k, v)$ is a t-(v, k, λ_t) design with $\lambda_t = 1$ and is also known as Steiner system. Kimberly (1971) has proposed that all resolvable 3-(v, k, λ_3) designs are affine resolvable iff they are 3-$(4\lambda_3 + 4, 2\lambda_{3+2}, \lambda_3)$ designs which are necessarily affine resolvable 3-designs. Also, Meitei (1992) had constructed a new series of resolvable 3-design starting from a 3-(v, k, λ_3) design having $b = 2(v-1)$, in which the newly constructed design has smaller block size than that of the starting design. In the literatures contributed by Hedayat and Kageyama (1980) and Kageyama and Hedayat (1983), many necessary conditions for the existence of t-design are available for different values of $t \geq 2$. In the same sprite, Wilson (1975) had developed many different conditions for the existence of t-design. Khosrovshahi and Tayfeh-Rezaie (2003) contributed some results on the existence of large sets of t-designs. McSorley and Soicher (2007) give a construction to obtain a t-design from a t-wise balanced design. Recently, Soicher (2011) also generalize the concept of "intersection numbers of order r" for t-designs proposed by Mendelsohn (1971) and show that analogous equations to those of Mendelsohn hold for generalized t-designs.

Many new parameters sets of t-designs on up to 40 points and $t \geq 7$ are given in Betten Laue and Wassermann (1999), using a computer software, DISCRETA. We interpret a theorem by Tran Van Trung (1986) from this point of view and see that from two t-designs with the parameters of a derived and a residual designs one obtains the third design derivable from the possibly existing (t+1)-design. Using the Kramer-Mesner method, by Acketa and Mudrinski (1996), 4-$(26, 6, \lambda)$ designs which is a special case for $t = 4$, have been proposed. In the literature proposed by Betten, Laue and Wassermann (1997) it is available that using derived design of t-design, a new t-design was constructed. Also, they presented 6-designs and 7-designs on 19 to 33 points. We start by presenting some notation and definitions used in this paper including defining exactly what we mean by a t-design.

Letting y_{ij}, μ, τ_i, β_j and e_{ij} be the observation receiving the ith treatment in the jth block, the general mean, the effect of the ith treatment, the effect of the jth block and the random error component of y_{ij} respectively. The general fixed effects linear additive model of a block design (v, b, r, k) under the assumptions (i) y_{ij}'s ~i.i.n.d.$(\mu+ \tau_i+ \beta_j, \sigma^2)$ and (ii) e_{ij} ~ i.i.n.d. $(0, \sigma^2)$, is given by $y_{ij}= \mu+ \tau_i+ \beta_j + e_{ij}$; i=1,2, …, t ; j=1, 2, …, b. ….. (1.1)

The estimation space and the error space are our prime interest to find such that the error space is to minimize so as our experiment-decision is genuine. As long as the error component, eij, is at minimum, the experiment-decision becomes more reliable. If the experimenter is permitted to have different models under the different assumptions, it is must to choose the model which can minimize the error component, eij. With the assumptions (i) yij's ~ i.i.n.d.(μ+ τ_i + βj + ρ_l, σ2) and (ii) eij ~ i.i.n.d. (0, σ2) under the resolvability of a block design (v, b, r, k), introducing a new component, known as, effect of the lth class, ρ_l, say,(l=1, 2, ..., t) the model (1.1) becomes yij = μ+ τ_i + β_j + ρ_l+ e_{ij}.…(1.2) Referring to the relations (1.1) and (1.2), it is clear that without disturbing y_{ij}, μ, τ_i, β_j in (1.1), we can minimize e_{ij} by using the relation (1.2) which shows the importance of the resolvability in reducing the error size.

II TERMS AND DEFINITIONS

Some terms and definitions are given in the following for future use.

Definition 2.1. Given a set S of v elements, if there are b subsets of S, each of size k, such that every t different elements occurs together in λ_t subsets , the whole system of the b subsets, treating each subsets as block, is called t-design where $0 < t \leq k$ and is denoted by t-(v, k, λ_t) design.

Denoting b and r as the block number and the replication number of the design, respectively then b=λ_0, and r= λ_1. Also $\binom{v}{t}$ λ_t = $\binom{k}{t}$b and (v-t+1) λ_t/ λ_{t-1} = (k-t+1). … (2.1)

Definition 2.2. A block design (v, b, r, k) is said to be affine resolvable if (i) the b blocks can be classified into t classes of the blocks, each of b/t=β, say, blocks (ii) each treatment gets replicated r/t =α, say, times in each class,(iii) any two blocks from the same class intersect with a fixed number of treatments, q_1, say, and (iv) any two blocks from the different classes intersect with a fixed number of treatments, q_2, say.

III MAIN RESULT ON v AND b (i.e. λ_0)

Suppose a t-(v, k, λt) design be an affine resolvable design. Accordingly, λ0 can be arranged in t classes, each of β bocks such that any two blocks belonged to the same class intersect q_1 treatments in common and any two blocks belonged to the different classes intersect q_2 treatments in common. Denoting Is identity matrix of order s and Js×p s×p-matrix containing only "1", then N′N = $A\otimes I_t + B\otimes(J_{t\times t} - I_t)$ where $A=kI_\beta + q_1 (J_{\beta\times\beta} - I_\beta)$, B= $q_2 J_{\beta\times\beta}$ and $\otimes$ is Kronecker product. By "$|E|$" denoting the determinant of a square matrix E, it clears that

$|N'N| = a|C|.|D|^{t-1}$ where $a=k+(\beta-1)q_1+(t-1)\beta q_2$,

$C=(k-q_1)I_{\beta-1}$, $D=(k-q_2)\otimes I_\beta+(q_1-q_2)\otimes(J_{\beta\times\beta}-I_\beta)$

$=\{k+q_1(\beta-1)+(t-1)\beta q_2\}(k-q_1)^{(\beta-1)t}\{k+(\beta-1)q_1 - \beta q_2\}^{t-1}$,

Since $|C| =(k-q_1)^{\beta-1}$,

$|D| = [(k-q_1)^{\beta-1}\{k+(\beta-1)q_1 - \beta q_2\}]^{t-1} = 0$, since $k+(\beta-1)q_1 - \beta q_2 = 0$, as $q_1 = k(\alpha-1)/(\beta-1)$,

$q_2 = k\alpha/\beta$.

So, N′N is a singular matrix of order b×b.

Then, rank(N′N)<b. … (3.1)

Similarly, $|NN'| =\{(v-1)\lambda_2+\lambda_1\}(\lambda_1-\lambda_2)^{v-1}$

$= \lambda_1 k(\lambda_1-\lambda_2)^{v-1}$ by (2.1)

$\neq 0$, since $\lambda_1\neq\lambda_2$.

Then, NN′ is a non-singular matrix of order v×v and consequently rank (NN′)=v … (3.2)

From (3.1) and (3.2), it is seen that v= rank (NN′)= rank(N)= rank(N′N)<b=λ_0.Thus, a result is immediate as follows.

Theorem 3.1 If a t-(v, k, λ_t) design is affine resolvable, then v< λ_0 (the number of blocks).

Remark 3.1 For t=2, the t-designs are BIBD and then our present finding given in the Theorem 3.1, holds good the Fisher's inequality v≤b on BIBD. However, the present finding shows a stronger statement "v<b" for t-design to be affine resolvable.

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