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MODEL BASED AND MODEL FREE COMBINATION CHARTS FOR CORRELATED DATA

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***Abstract*: When significant autocorrelation is present in the data , traditional charts cannot be applied directly to the data. In such situations either control charts are modified to accommodate correlation or an appropriate time series model is fitted for the data and the residuals from the model are monitored with traditional charts. In this paper model based and model- free combination charts for simultaneous monitoring of mean and variance of autocorrelated data are discussed and their performance is evaluated and compared based on their ARL**

Keywords: *Autocorrelation, forecast residuals, ARL, AR(1) process*

I INTRODUCTION

It is not advisable to apply traditional charts directly to the data without any modification when there is significant autocorrelation in the data. Two general approaches have been considered in the literature for dealing with autocorrelated data. In the first approach, control limits of the traditional charts are modified to accommodate autocorrelation in the data. This method can be adopted when the autocorrelation is low. In the second approach, a time series model is fitted to the data and the forecast errors (residuals) from the fitted model are monitored using traditional charts. But when control charts are applied to residuals, only a fraction of the shift that has occurred in the process mean level will be transferred to the residual means due to forecast recovery. As a result the performance of the chart will be deteriorated. Simultaneous monitoring of mean and variance using separate charts also shows lowered performance when autocorrelation is present in the data [1]. Positive autocorrelation result in an increased average false alarm rate for a location chart and decreased false alarm rate for a variance chart. Other than the time series model based methods, certain model free methods are also developed in the literature for the problem of monitoring autocorrelated data. Some of the model free and model based joint monitoring schemes are discussed in the following sessions.

II MODEL BASED JOINT MONITORING SCHEMES

Various approaches to joint monitoring of mean and variance of autocorrelated processes in a single chart are considered in the literature. A few of them are mentioned by Cheng and Thaga [2]. Lu and Reynolds [3] discuss both schemes in which procedures for parameter estimation and control limit selection are adjusted to account for the autocorrelation and schemes in which a time series model is utilized.

Lu and Reynolds [4] evaluate six different combination charting. They consider an AR(1) process with an additional random error. EWMA chart of observations and EWMA chart of residuals are used for monitoring the mean. Four charts are compared for their effectiveness in monitoring the process variance. The first chart is the EWMA chart of log of squared residuals, the second is the Shewhart chart of the squared residuals, which is equivalent to a Shewhart chart of residuals. The third chart is the traditional moving range chart. The fourth is the EWMA chart of residuals which is included in the comparison to study how a chart designed primarily to detect a change in the mean will respond to an increase in variance. It is shown that the combination consisting of an EWMA chart of the observations and a Shewhart chart of residuals had the best performance for processes with moderate or low autocorrelation.

The MAX – CUSUM Chart proposed by Cheng and Thaga [2] assumes an AR(1) process with an additional random error [4]. The present study discuss and compare the performance of the combination schemes (i) EWMA chart of the observations and a Shewhart chart of residuals (ii) EWMA of the residuals and EWMA of log of squared residuals [4] and (iii) the distribution free approach proposed by Runger and Willemain [5] for an AR(1) process.

III MODEL FREE MONITORING SCHEMES

Distribution based process control schemes require the in control underlying process to follow a specific probability distribution model, or certain characteristics of the process , such as the correlation structure to be known. These charts are often criticized for the fact that the underlying assumption may be violated, resulting in performance deterioration and also for the fact that their control limits are determined by trial and error which is sometimes inconvenient. To overcome this limitation a few distribution free methods are proposed in the literature. Johnson and Bagshaw [6] Runger and Willemain [5] Kim *et.al* [7] discuss distribution free methods for monitoring the process mean.

Batch Means Charts

Unweighted batch mean (UBM) chart is proposed as an alternative to time-series modeling [5]. Since it requires no time series modeling, it is considered as a model free approach. The UBM chart plots arithmetic average of successive observations and exploits the large number of observations available in a data rich environment. The averaging of the observations dilutes the autocorrelation.

The j th unweighted batch mean is given by

$$V_j = \frac{\sum_{i=1}^{b} Y_{(j-1)b+i}}{b} \qquad j=1,2,..... \qquad (1)$$

Procedures for determining the appropriate batch size b is given in Law and Carson [5, 8].

Tabular CUSUM Chart

Kim [9] discusses distribution free method for controlling variability of autocorrelated process. The proposed Distribution Free Tabular CUSUM (DFTCV) Chart is obtained by combining a distribution free variance estimation technique with a distribution free SPC chart for monitoring the mean. The time series variance estimator CvM which requires only one batch of observations is used to get an estimate. If V_i denote the CvM variance estimate from the i^{th} batch, then the problem of monitoring the variability of the actual observations Y_1, Y_2, …. becomes that of monitoring the mean of V_1, V_2, ….. Therefore by combining a distribution free variance estimation technique with a distribution free chart for monitoring mean one can obtain a distribution free scheme for monitoring variability.

Scale – Rank Chart

Das and Bhattacharya [10] adapted a nonparametric two-sample test for dispersion. The chart statistic is based on the absolute deviations of each observation from a sample median which is obtained from an in-control sample. The absolute deviations are ranked and squared. The sum of the squared ranks of one sample is taken as the chart statistic. Control limits for the scheme are ± 3. Jones-Farmer and Champ [11] also discuss about scale rank chart in which a sample of m sub groups of size n is taken from the process.

IV COMPARISON OF MODEL BASED AND MODEL-FREE MONITORING SCHEMES

In this study four monitoring schemes are evaluated and compared based on their ARL performance. The four monitoring schemes are

(1) EWMA chart of the observations and a Shewhart chart of residuals
(2) EWMA of the residuals and EWMA of log of squared residuals
(3) UBM Chart for observations
(4) UBM Chart for forecast residuals

Time Series model

This study assumes that an AR (1) model for the process observations which is special case of Autoregressive Integrated Moving Average (ARIMA) model. The model is given by

$$X_t - \mu = \varphi\,(X_{t-1} - \mu) + \varepsilon_t$$

where φ is the autoregressive parameter, $|\varphi| < 1$ and ε_t is a sequence of independent and identically distributed normal random errors with mean 0 and variance $\sigma_\varepsilon{}^2$.

If the model parameters are estimated correctly, then the forecast residuals

$$e_t = X_t - \hat{\mu}\,(1-\hat{\varphi}) - \hat{\varphi} X_{t-1}$$

are i.i.d normal with mean 0 and variance σ_e^2 which approximate ε_t. The one step ahead forecast is given by

$$\hat{X}_t = \mu(1-\varphi) + \varphi X_{t-1}$$

Consider a shift of δ in the process mean level between time t-1 and t, then the sequence of residuals will be

$$e_t = X_t - \hat{X}_t$$
$$= [\mu + \delta + \varphi\,(X_{t-1} - \mu) + \varepsilon_t] - [\,\mu(1-\varphi) + \varphi X_{t-1}]$$
$$= \delta + \varepsilon_t$$
$$e_{t+1} = X_{t+1} - \hat{X}_{t+1}$$

$= [\mu + \delta + \varphi\,(X_t - (\mu + \delta) + \varepsilon_{t+1}] -$

$[\mu(1-\varphi) + \varphi X_t]$

$= \delta\,(1-\varphi) + \varepsilon_{t+1}$

In general, if the mean shift to $\mu + \delta$ between t = r-1 and r and it is assumed to shift no further then we have

$$e_t = \varepsilon_t \qquad t = 1,2,\ldots, r\text{-}1$$

$$= \delta + \varepsilon_t \qquad t = r$$

$$= \delta\,(1-\varphi) + \varepsilon_t \qquad t = r+1, r+2, \ldots\ldots\ldots\ldots$$

Note that the same amount of shift is reflected in the time point immediately following the shift and then it reduce to δ $(1-\varphi)$ due to forecast recovery phenomenon. As the autoregressive parameter nears 1 chance of detecting the shift after the initial time point become negligible.

EWMA Chart of the Observations and Individual Chart of Residuals

The combination of EWMA chart of the observations and individual chart of residuals is selected since it gives reasonably good overall performance [4].

Let X_t represents the observation taken at time point t. The EWMA chart for the original observations is based on the control statistic,

$$Y_t = (1-\lambda)Y_{t-1} + \lambda X_t \qquad t = 1, 2, \ldots\ldots \quad (2)$$

where λ is a smoothing constant satisfying $0 < \lambda \leq 1$. The control limits for the chart are

$$\mu_0 \pm h \quad \sigma_0 \sqrt{\lambda/(2-\lambda)} \qquad (3)$$

where $\sigma_0 \sqrt{\lambda/(2-\lambda)}$ is the in- control standard deviation of Y_t. Y_t is initialised as $Y_0 = 0$ and *h* is chosen to get a desired ARL_0. λ is taken as 0.2.

For monitoring the process variance, the chart considered is Shewhart Individual chart of squared residuals which is equivalent to Shewhart Individual chart of residuals. The statistic plotted at time t is

$$e_t = X_t - \hat{X}_t \qquad (4)$$

and the control limits are $\pm h\sigma_e$.

EWMA of the residuals and EWMA of log of squared residuals

Let X_t denote the observation taken at time point t. The EWMA chart for the forecast residuals for monitoring the process mean is based on the control statistic,

$$Z_t = (1-\lambda)Z_{t-1} + \lambda e_t \qquad t = 1, 2, \ldots\ldots \qquad (5)$$

where e_t is given by (4) and λ = .2. The control limits are given by

$$\pm h \quad \sigma_e \sqrt{\lambda/(2-\lambda)} \qquad (6)$$

For monitoring the process variance Crowder and Hamilton [12] suggested the EWMA chart of logs of the sample variances. In the present study, an EWMA chart based on the logs of the squared residuals [4] is used for monitoring mean.

The control statistic of EWMA chart based on the logs of the squared residuals is

$$U_t = max\{(1-\lambda)U_{t-1} + \lambda\, ln(e_t^2), \ln(\sigma_0^2)\} \qquad (7)$$

The initial value of the EWMA statistic $U_0 = \ln(\sigma_0^2)$. When the process is in control $\sigma_e^2 = \sigma_0^2$.

The EWMA chart considered above is one sided. The control statistic resets to the target $\ln(\sigma_0^2)$ at the next sample whenever the statistic drops below this target.

The residuals e_t are normally distributed with mean zero, it follows that e_t^2 has gamma distribution with location parameter ½ and scale parameter $1/var\,({e_t}^2)$. It is known that the scale parameter in the gamma distribution becomes the location parameter in the log gamma distribution. Therefore, it is reasonable to use the logs of the squared residuals in the EWMA chart to monitor shifts in the process variance.

The in-control mean and variance of $ln(e_t^2)$ can be obtained [4] as

$$E(ln(e_t^2)) = \ln(\sigma_0^2) - 6/5 \qquad (8)$$

$$\text{and} \qquad Var\,(ln(e_t^2)) = 64/15 \qquad (9)$$

Then the control limits for the EWMA chart of the logs of the residuals is

$$\ln(\sigma_0^2) + h\sqrt{\frac{64\lambda}{15(2-\lambda)}} \qquad (10)$$

where *h* is determined to get desired ARL_0.

UBM Chart for Observations

The model free approach for monitoring autocorrelated data, UBM chart is also considered for comparison. If X_t denote the observation at time t then, the UBM statistic plotted at t is

$$V_t = \frac{\sum_{i=1}^{b} x_{(t-1)b+i}}{b} \qquad for\ t = 1,2,\ldots\ldots \qquad (11)$$

The control limits of the chart are

$$\mu \pm h\sigma \qquad (12)$$

UBM Chart for forecast residuals

If X_t denote the observation at time t and if e_t denote the forecast residual then the UBM chart of forecast residuals plot at time t ,

$$R_t = \frac{\sum_{i=1}^{b} e_{(t-1)b+i}}{b} \qquad for\ t = 1,2, \ldots \ldots \qquad (13)$$

The forecast residual e_t is obtained using eqn. (4). The control limits of the chart are

$$\pm h\sigma_e \qquad (14)$$

V DESCRIPTION OF THE SIMULATION

A comparative study for evaluating the performance of the four schemes discussed above is made based on ARL of the schemes applied to simulated observations. In this study, the in-control ARL of all the schemes were made equal to 185. The control chart parameters of all the charts are selected to get the desired in control ARL of 185 for the combination charting. Without loss of generality we assume that the process mean $\mu = 0$. ε_t s are assumed to be Normal with mean zero and variance $\sigma_\varepsilon{}^2 = 1$. The autoregressive parameter φ is allowed to vary from 0 to .95. The process is allowed to be in control during the initial period and a shift is introduced at the 51st observation. Shifts in both mean and variance are considered. Process mean is allowed to shift from 0 to $3\sigma_\varepsilon$ in increments of $0.5\sigma_\varepsilon$. The variance of ε_t is allowed to vary from $1\sigma_\varepsilon{}^2$ to $3\sigma_\varepsilon{}^2$. The value of λ is chosen as 0.20 for the EWMA charts. The control limits of the charts are calculated using the equations (3), (6), (10), (12) and (14)

Performance evaluation of the schemes are made based on the simulation study conducted according to the following steps.

Step1: N(0,1) random numbers ε_t are generated using R.

Step2: The observations X_t form an AR(1) model are obtained using equations (6.4) for φ ranging from 0 to 0.95

Step3: The first 50 ε_ts are discarded to allow the time series to stabilize.

Step4: The one step ahead forecast for X_{t+1} is obtained using equation(6.6)

Step5: Forecast residual e_t is obtained using (6.10)

Step6: The original observations / forecast residuals are monitored using the four monitoring schemes and the run length of each control schemes are recorded.

Step7: Steps (1) to (4) are repeated 10,000 times and the ARLs are calculated.

Step8: Steps (1) to (7) are repeated for process mean shifting from μ to $\mu+3\sigma$ in increments of $.5\sigma$ and process variance moving from σ to 3σ in increments of 1σ

VI OBSERVATIONS AND CONCLUSION

Four schemes discussed above were considered in the study but the run lengths of three schemes are only given in the table since the UBM chart directly applied to original observation showed inferior performance to all the other schemes for all magnitude of shifts and for every choice of autoregressive parameter.

Table 1 shows simulated ARL values when mean and variance increase from the in control values for $\varphi = 0, .25, .5, .75, and\ .95$. $\varphi = 0$ corresponds to i.i.d observations. From the table it can be observed that the UBM chart for residuals is uniformly superior in performance to the other two combined schemes. This indicates that the UBM chart which is designed to monitor the process mean level is also efficient in monitoring shifts in process variance. Figure 8 shows the ARL values when there is an increase of magnitude $\delta\sigma$ in the process mean level for $\varphi = 0,0.5,0.95$. It can be observed from the figure that the combination of EWMA for residuals and EWMA for log of squared residuals and EWMA for original observations and Shewhart Individual for squared residuals are almost equal in their performance. But the EWMA Individuals combination is slightly better in their performance except for very high level of autocorrelation. Even when autocorrelation is very high for large magnitude of shifts it perform better that the EWMA for residuals and EWMA for log of squared residuals combination.

When there is increase in variance all the three schemes shows equally good performance irrespective of the magnitude of autocorrelation.

			λ				
ϕ	Control Scheme	δ	1	1.5	2	2.5	3
0	$EWMA_{res}$ & $EWMA\ ln(res^2)$	0	185.87	15.86	6.84	4.5	3.49
	$EWMA_{obs}$ & $Individuals_{res}$		185.27	16.56	6.47	4.06	3.06
	UBM for residuals		185.26	15.49	6.03	3.73	2.8
	$EWMA_{res}$ & $EWMA\ ln(res^2)$	0.5	32.06	11.78	6.35	4.37	3.45
	$EWMA_{obs}$ & $Individuals_{res}$		31.75	11.6	5.89	3.87	2.95
	UBM for residuals		11.72	5.5	3.71	3	2.5
	$EWMA_{res}$ & $EWMA\ ln(res^2)$	1	9.39	7.06	5.19	3.94	3.27
	$EWMA_{obs}$ & $Individuals_{res}$		9.12	6.69	4.69	3.44	2.79
	UBM for residuals		1.92	1.95	1.94	1.93	1.87
	$EWMA_{res}$ & $EWMA\ ln(res^2)$	1.5	5.05	4.76	4.08	3.49	3.02
	$EWMA_{obs}$ & $Individuals_{res}$		4.81	4.25	3.55	2.97	2.55
	UBM for residuals		1.08	1.19	1.3	1.39	1.45
	$EWMA_{res}$ & $EWMA\ ln(res^2)$	2	3.47	3.46	3.34	3.03	2.73
	$EWMA_{obs}$ & $Individuals_{res}$		3.16	3.03	2.81	2.54	2.29
	UBM for residuals		1	1.03	1.08	1.14	1.2
	$EWMA_{res}$ & $EWMA\ ln(res^2)$	2.5	2.67	2.75	2.76	2.6	2.47
	$EWMA_{obs}$ & $Individuals_{res}$		2.28	2.27	2.26	2.17	2.08
	UBM for residuals		1	1	1.01	1.05	1.08
	$EWMA_{res}$ & $EWMA\ ln(res^2)$	3	2.23	2.31	2.33	2.29	2.25
	$EWMA_{obs}$ & $Individuals_{res}$		1.72	1.79	1.87	1.88	1.85
	UBM for residuals		1	1	1	1.01	1.03
0.25	$EWMA_{res}$ & $EWMA\ ln(res^2)$	0	185.74	16.03	6.97	4.48	3.5
	$EWMA_{obs}$ & $Individuals_{res}$		185.18	16.43	6.51	4.04	2.98
	UBM for residuals		185.27	15.81	6.08	3.79	2.82
	$EWMA_{res}$ & $EWMA\ ln(res^2)$	0.5	52.31	13.21	6.52	4.47	3.46
	$EWMA_{obs}$ & $Individuals_{res}$		51.59	13.4	6.16	3.92	2.95
	UBM for residuals		23.83	7.6	4.55	3.25	2.62
	$EWMA_{res}$ & $EWMA\ ln(res^2)$	1	15.6	9.03	5.8	4.22	3.37
	$EWMA_{obs}$ & $Individuals_{res}$		15.25	8.73	5.29	3.69	2.87
	UBM for residuals		3.97	3.03	2.64	2.39	2.14
	$EWMA_{res}$ & $EWMA\ ln(res^2)$	1.5	7.73	6.32	4.91	3.84	3.2
	$EWMA_{obs}$ & $Individuals_{res}$		7.54	5.89	4.38	3.33	2.74
	UBM for residuals		1.53	1.66	1.72	1.76	1.74
	$EWMA_{res}$ & $EWMA\ ln(res^2)$	2	5.06	4.73	4.14	3.47	3.01
	$EWMA_{obs}$ & $Individuals_{res}$		4.82	4.24	3.61	3	2.55
	UBM for residuals		1.08	1.19	1.3	1.39	1.44
	$EWMA_{res}$ & $EWMA\ ln(res^2)$	3	3.01	3.1	3.02	2.83	2.57
	$EWMA_{obs}$ & $Individuals_{res}$		2.67	2.56	2.48	2.34	2.18
	UBM for residuals		1	1.01	1.04	1.09	1.12

Table 1: ARL of the schemes applied to AR (1) process when the autoregressive parameter φ moves from 0 to .95 for a shift δσ in process mean level and when the variance increase to λσ.

φ	Control Scheme	δ	λ = 1	1.5	2	2.5	3
0.5	$EWMA_{res}$ & EWMA $ln(res^2)$	0	185.35	15.83	6.91	4.49	3.46
	$EWMA_{obs}$ & $Individuals_{res}$		185.78	16.64	6.52	4.04	2.99
	UBM for residuals		185.48	15.88	6.11	3.76	2.82
	$EWMA_{res}$ & EWMA $ln(res^2)$	0.5	89.94	14.6	6.75	4.42	3.45
	$EWMA_{obs}$ & $Individuals_{res}$		88.49	15.05	6.35	3.97	2.96
	UBM for residuals		51.07	10.78	5.26	3.55	2.7
	$EWMA_{res}$ & EWMA $ln(res^2)$	1	32.24	11.91	6.3	4.4	3.38
	$EWMA_{obs}$ & $Individuals_{res}$		31.56	11.65	5.92	3.88	2.93
	UBM for residuals		11.71	5.46	3.81	2.97	2.48
	$EWMA_{res}$ & EWMA $ln(res^2)$	1.5	15.23	9.08	5.72	4.16	3.36
	$EWMA_{obs}$ & $Individuals_{res}$		15.2	8.82	5.24	3.63	2.84
	UBM for residuals		3.92	3.01	2.63	2.38	2.16
	$EWMA_{res}$ & EWMA $ln(res^2)$	2	9.38	7.13	5.16	3.98	3.21
	$EWMA_{obs}$ & $Individuals_{res}$		9.11	6.64	4.63	3.47	2.77
	UBM for residuals		1.91	1.95	1.96	1.93	1.87
	$EWMA_{res}$ & EWMA $ln(res^2)$	3	5.07	4.71	4.11	3.46	3
	$EWMA_{obs}$ & $Individuals_{res}$		4.79	4.24	3.55	2.96	2.54
	UBM for residuals		1.07	1.2	1.3	1.37	1.44
0.75	$EWMA_{res}$ & EWMA $ln(res^2)$	0	185.48	16.05	6.8	4.53	3.5
	$EWMA_{obs}$ & $Individuals_{res}$		185.2	16.46	6.51	4.03	2.96
	UBM for residuals		185.03	15.68	6.13	3.76	2.84
	$EWMA_{res}$ & EWMA $ln(res^2)$	0.5	147.9	15.68	6.87	4.51	3.46
	$EWMA_{obs}$ & $Individuals_{res}$		145.61	16.04	6.52	3.95	3.02
	UBM for residuals		119.39	14.15	5.97	3.69	2.76
	$EWMA_{res}$ & EWMA $ln(res^2)$	1	89.28	14.67	6.83	4.57	3.48
	$EWMA_{obs}$ & $Individuals_{res}$		86.63	15.13	6.32	3.94	2.99
	UBM for residuals		51.89	10.45	5.22	3.57	2.73
	$EWMA_{res}$ & EWMA $ln(res^2)$	1.5	52.12	13.2	6.58	4.46	3.46
	$EWMA_{obs}$ & $Individuals_{res}$		51.93	13.61	6.17	3.93	2.96
	UBM for residuals		23.48	7.65	4.52	3.25	2.62
	$EWMA_{res}$ & EWMA $ln(res^2)$	2	32.41	11.83	6.38	4.36	3.42
	$EWMA_{obs}$ & $Individuals_{res}$		31.38	11.69	5.87	3.86	2.95
	UBM for residuals		11.82	5.46	3.74	2.93	2.44
	$EWMA_{res}$ & EWMA $ln(res^2)$	3	15.49	9.16	5.75	4.12	3.38
	$EWMA_{obs}$ & $Individuals_{res}$		15.11	8.77	5.32	3.68	2.83
	UBM for residuals		3.95	3.02	2.63	2.37	2.16
0.95	$EWMA_{res}$ & EWMA $ln(res^2)$	0	185.67	15.95	6.88	4.56	3.49
	$EWMA_{obs}$ & $Individuals_{res}$		185.41	16.57	6.59	4.06	2.99
	UBM for residuals		185.71	15.91	5.99	3.75	2.83
	$EWMA_{res}$ & EWMA $ln(res^2)$	0.5	180.4	15.97	6.92	4.55	3.48
	$EWMA_{obs}$ & $Individuals_{res}$		184.05	16.93	6.59	4.03	3.04
	UBM for residuals		182.12	15.49	6	3.77	2.85
	$EWMA_{res}$ & EWMA $ln(res^2)$	1	177.63	15.94	6.89	4.51	3.54
	$EWMA_{obs}$ & $Individuals_{res}$		176.69	16.49	6.5	4.03	2.98
	UBM for residuals		171.87	15.28	5.98	3.74	2.77
	$EWMA_{res}$ & EWMA $ln(res^2)$	1.5	167.61	15.88	6.81	4.59	3.57
	$EWMA_{obs}$ & $Individuals_{res}$		171.86	16.18	6.51	4.01	2.98
	UBM for residuals		154.08	15.19	5.99	3.75	2.81
	$EWMA_{res}$ & EWMA $ln(res^2)$	2	157.38	15.71	6.83	4.5	3.49
	$EWMA_{obs}$ & $Individuals_{res}$		159.71	16.39	6.59	3.96	2.97
	UBM for residuals		136.69	14.5	5.99	3.73	2.78
	$EWMA_{res}$ & EWMA $ln(res^2)$	3	134.44	15.4	6.92	4.52	3.51
	$EWMA_{obs}$ & $Individuals_{res}$		133.1	16.13	6.52	4	2.97
	UBM for residuals		99.99	13.43	5.78	3.63	2.78

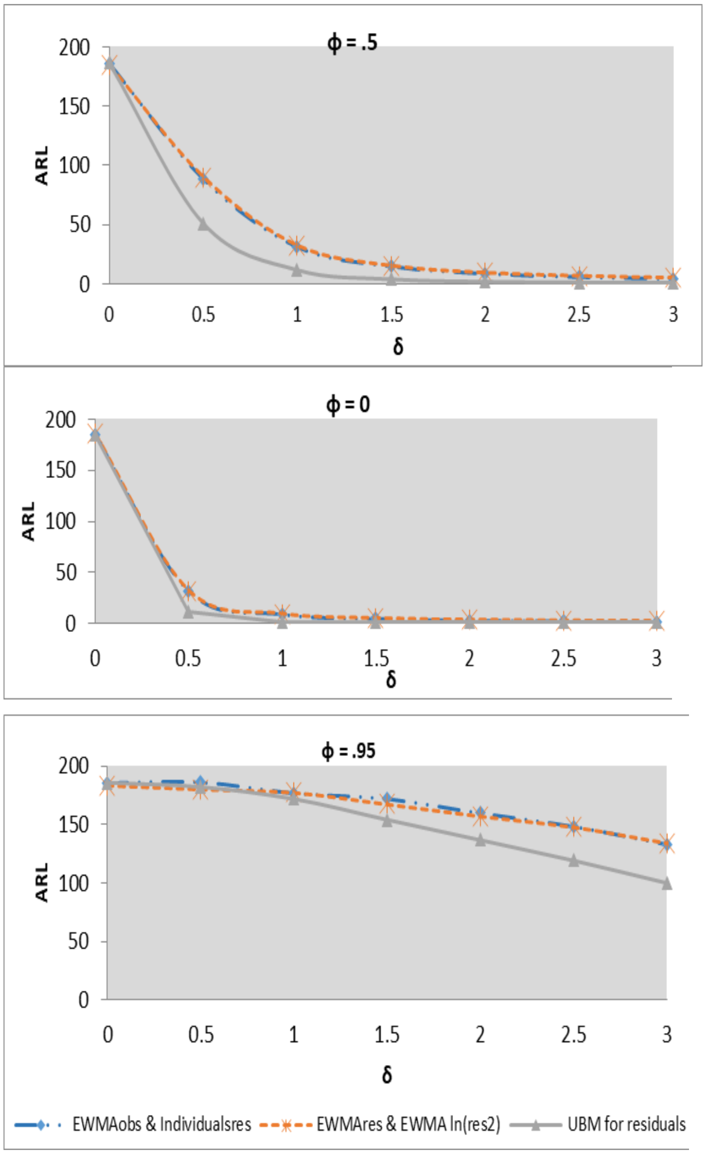

Figure 1 : ARL of the control schemes applied to AR(1) process for $\varphi = 0, 0.5, 0.95$ when there is shift of magnitude $\delta\sigma$ in the process mean level.

REFERENCES

1. Cherian, A (2021) Effect of Autocorrelation on Combined Monitoring Schemes Journal of Xi'an University of Architecture & Technology, Volume XIII, Issue 3, 38 - 46
2. Cheng, S. W. and Thaga, K. (2006). Single Variables Control Charts: An Overview. Quality and Reliability Engineering International, 22(7), 811-820.
3. Lu, C.W. and Reynolds, M.R.,Jr. (1999b) "Control Charts for Monitoring the Mean and Variance of Autocorrelated Processes." Journal of Quality Technology, 31(3), 259-274.
4. Lu, C.W. and Reynolds, M.R.,Jr. (1999a) "EWMA Control Charts for Monitoring the Mean of Autocorrelated Processes." Journal of Quality Technology, 31, 166-188.
5. Runger, G.C. and Willemain, T.R. (1995) "Model-Based and Model-Free Control of Autocorrelated Processes", Journal of Quality Technology, 27(4), 283-292.
6. Johnson, R.A and Bagshaw, M(1974) " The effect of Serial Correlation on the Performance of CUSUM Tests", Technometrics, 16, 103-112
7. Kim S.H.C, Alexopoulos, K,L, Tsui, and J.R Willson (2006) " A distribution Free tabular CUSUM Chart for Autocorrelated Data", IEE Transactions, 231-237
8. Law, A. and Carson, J.S. (1979). "A Sequential Procedure for Determining the Length of Steady-State Simulation". Operations Research, 29, 1011-1025.
9. Kim (2006) " Monitoring Variability of Autocorrelated Processes Using Standardized Time Series Variance Estimators", Proceedings of the 2006 Winter Simulation Conference , 231-237
10. Das, N and Battacharya, A(2008) " A New non-parametric Control Chart for Controlling Variability", Quality Technology and Quantitative Management, 5(4), pp. 351-361
11. Jones-Farmer and Champ (2010) " A Distribution – Free Phase I Control Chart for Subgroup Scale" , Journal of Quality Technology., 42(4), 373 - 387.
12. Crowder, S.V and Hamilton, M.D (1992) " An EWMA for monitoring a Process Standard Deviation", Journal of Quality Technology, 24,155-162.