|| Volume 6 || Issue 6 || June 2021 || ISO 3297:2007 Certified ISSN (Online) 2456-3293

OPEN ACCESS INTERNATIONAL JOURNAL OF SCIENCE & ENGINEERING

FREE VIBRATION ANALYSIS OF LAMINATED COMPOSITE PLATES WITH CUTOUT USING FINITE ELEMENT METHOD

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Abstract: It is evident that substantial work has been done to understand the free vibration response of solid composite laminated plates with cutout for different combinations of parameters that affect the dynamic characteristics of plates. In case of square plates with square or rectangular cutouts, a complete picture on dynamic characteristics for different layup sequences and different boundary conditions are not available. Again the difference in dynamic behavior of laminated composite plates with cut-out for various thicknesses are not covered previously. The objective of the present investigation is to understand the effect of the rectangular and square shaped cutout on the free vibration response of laminated composite plate based on different boundary conditions, layup sequences and thickness.

Keywords: Classical Lamination Theory(CLT),First Order Shear deformation theory(FSDT),High Order Shear Deformation Theory(HSDT),Layer Wise Lamination theory(LLT)

I INTRODUCTION

In general, a material that is formed by combining different materials on a macroscopic scale is known as composite material. A composite material usually derives its properties from its constituents. Plywood and reinforced concrete are some of the composite materials, which are being used for along time. A composite material may be classified as fibrous, laminated and particulate. In fibrous composite materials, the fibers are embedded in the matrix. The load is mainly carried by the fibers. The matrix binds the fibers and distributes the load among the fibers and prevents the fibers from direct exposure to the environment. The fibers and the matrix may be of the same material or different materials. The fibers used in these material are characterized by their near crystal size diameter. In laminated composite materials, layers of different properties are bonded together to act as an integral part. In particulate composite materials, particles of differing materials are held in a matrix.

Nowadays, fiber reinforced plastics are being increasingly used in aerospace applications due to their high specific strength, high specific stiffness and low density. In addition, they have good corrosive resistance. A designer can easily tailor these materials for different applications. Epoxy, Polyester, Vinyl- Ester, Phenolic are commonly employed as a matrix. High performance thermoplastics are also being utilized on a large scale. It is generally considered that the reduction in weight upto 25% can be achieved by using fiber reinforced plastics in case of conventional materials of an aircraft. Glass-Epoxy composite materials were the first to be used in aircraft structures in mid forties. Due to low specific stiffness of Glass- Epoxy, compared to conventional aircraft materials, it was not used in major applications. In around 1960 Graphite and Boron fibers were developed. Since, Graphite- Epoxy and Boron-Epoxy composite materials are superior to conventional metals used in aircrafts, in terms of both strength and stiffness, they were used in aircraft structural applications to a significant level.

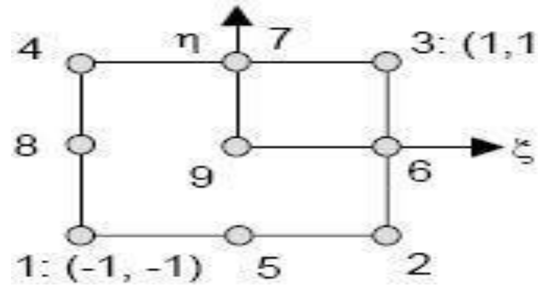
II.LITERATURE REVIEW

Free vibration of laminated composite plate with cutout :

References dealing with free vibration response of composite plates with cutouts are relatively less in number. Paramasivam [19] proposed a method to determine the effect of square openings on the fundamental frequency of square isotropic plates for different boundary conditions using the finite difference method. Results were obtained for simply supported and clamped boundary conditions. Aksu and Ali [20] developed a theory to study the dynamic characteristics of isotropic and orthotropic rectangular plates with one or two rectangular cutouts. They employed a method based on the use of variational principles in conjunction with finite difference technique. Rajamani and Prabhakaran [21,22] studied the effect of a centrally located square cutout on the natural frequencies of square, simply supported and clamped symmetrically laminated composite plates for free and forced vibration cases. They assumed that the effect of cutout is equivalent to an external loading. Some of the conclusions drawn were: cutouts make plates less stiff for medium cutout sizes for all modulus ratios; the fundamental frequency increases with the increase in fiber orientation for all cutout parameters and for all modulus ratios; higher modes interchange for all modulus ratios except in few cases; the fundamental frequency increases with the cutout ratio for the 450 fiber orientation only for all modulus ratios. Ali and Atwal [23] presented a simplified method, based on Rayleigh's principle, for the dynamic analysis of plates with rectangular cutouts. Results were obtained for a simply supported square plate with square and rectangular openings of selected size. Reddy [24] studied large amplitude flexural vibrations of rectangular plates, using a finite element formulation based on the Reissner– Mindlin theory in conjunction with non-linear (large rotation) strain displacement relations. Numerical results dealing with the effect of parameters like side-to-thickness ratio, plate side-to-cutout side ratio and anisotropy were presented for both linear as well as non-linear frequencies. It was shown that the effect of shear deformation is more pronounced in the case of clamped plates than in simply supported plates. The effect of transverse shear was found to be more pronounced on higher modes than on fundamental mode. Lee et al. [25] presented a numerical method based on the Rayleigh principle and using CLPT for obtaining natural frequencies of a simply supported composite rectangular plate having a central rectangular cutout and double square cutouts. Chang and Chiang [26] used Hamilton's variational principle, the Mindlin plate theory and finite element method to study the effect of boundary.

III.ISOPARAMETRIC FORMULATION :

The isoparametric means "same parameters" and is applied here because same interpolation functions are used to interpolate the Magnitude of co-ordinates as well as the degree of freedom. If the interpolation functions for the degrees of freedom are of higher order than those used for co-ordinate interpolations, the element is called subparametric, while the reverse is the case for superparametric elements. By the use of isoparametric elements it is easier to simulate complicated geometries, while simplifying the computations simultaneously. A flat Mindlin- plate element, in general, has five displacement degrees of freedom at each node as shown in Eq. (34). Nine noded Lagrangian element [Fig 6.] with five degree of freedom at each node, i.e., u0, v0, w, θx, θy is considered for finite element formulation.



Nine noded Lagrangian element

The co-ordinates and the elastic parameters inside an element can be interpolated using the shape functions and the nodal values as follows: (57)

$$u_i = \sum_{j=1}^{NEL} N_j(\xi,\eta)(d_i)_j;\ i = 1,2,3,4,5$$

Here, NEL stands for nodes per element, whereas, ui stands for the five intra-element displacement components uo, vo, wo, θx, and θy at node i while di are the corresponding nodal displacements at node i, both considered at the middle surface of the plate.

The shape functions Nj in equations (58) are defined as

Nj = (ξ2 + ξξj)(η2 + ηηj)/4; j = 1,2,3,4

Nj = η 2(η2 – ηη) (1 − ξ2)/2 + ξ 2(ξ2 – ξξ)(1 − η2)/2; j = 5,6,7,8

Nj = (1 − ξ2) (1 − η2); j = 9 (58)

Where ξ and η are the local natural co-ordinates of the element and ξj and ηj are the value of them at node j.The strains at the mid-plane may be similarly written by taking the derivatives of the shape functions with respect to the spatial co-ordinates as shown below:

$$\{\varepsilon^0\} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \sum_{j=1}^{NEL} \left[\begin{bmatrix} N_{j,x} & 0 & 0 & 0 & 0 \\ 0 & N_{j,y} & 0 & 0 & 0 \\ N_{j,y} & N_{j,x} & 0 & 0 & 0 \\ 0 & 0 & 0 & N_{j,x} & 0 \\ 0 & 0 & 0 & 0 & N_{j,y} \\ 0 & 0 & 0 & N_{j,y} & N_{j,x} \\ 0 & 0 & N_{j,y} & 0 & N_j \\ 0 & 0 & N_{j,x} & N_j & 0 \end{bmatrix} \begin{Bmatrix} u_{oj} \\ v_{oj} \\ w_{oj} \\ \theta_{xj} \\ \theta_{yj} \end{Bmatrix} \right]$$

Or $\{\varepsilon^0\} = [B]\{d\}$.

Since the shape functions, the elastic matrix, $[D]$, and the inertia matrix, $[\rho]$, are already known, as cited in Fig. (6) and Eqs. (39) and (52), the element stiffness and mass arrays can be easily computed in local axes using Eqs. (53), (54) and (55). These element arrays are next transformed to fit the global axes before they are properly assembled. In the next section the transformation matrix is formed and the transformation to be applied over the element matrices are summarily discussed for completeness.

The derivatives of $\frac{1}{2}$ the shape functions N_j with respect to x and y are expressed in terms of derivatives with respect to ξ and η by the following relationship:

$$\begin{Bmatrix} N_{j,x} \\ N_{j,y} \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} N_{j,\xi} \\ N_{j,\eta} \end{Bmatrix}$$

Where, $[J] = \begin{bmatrix} x_{,\xi} & y_{,\xi} \\ x_{,\eta} & y_{,\eta} \end{bmatrix}$

The Principle of Total Potential Energy described in sec. 4.2.4 is applied for the element to derive the stiffness and mass matrices.

III.2. Element Stiffness Matrix :

The potential energy of deformation for the element, given by equation (41), is

$$U = \frac{1}{2}\iint_{Ae}\{\varepsilon\}^T [D]\{\varepsilon\} dA$$

if $\{\varepsilon\} = [B]\{\delta_e\} = [[B_1]\,[B_2]\,\ldots\ldots\ldots[B_9]]\,\{\delta_e\}$

where $\{\varepsilon\} = \{\varepsilon_x\ \varepsilon_y\ \varepsilon_{xy}\ \kappa_x\ \kappa_y\ \kappa_{xy}\ \varepsilon_{xz}\ \varepsilon_{yz}\}^T$

where $\{\varepsilon\} = \{\varepsilon_x\ \varepsilon_y\ \varepsilon_{xy}\ \kappa_x\ \kappa_y\ \kappa_{xy}\ \varepsilon_{xz}\ \varepsilon_{yz}\}^T$

$\{\delta_e\} = \{u_{01}, v_{01}, w_1, \theta_{x1}, \theta_{y1}\ \ u_{09}, v_{09}, w_9, \theta_{x9}, \theta_{y9}\}$

$$[B_i] = \ (i = 1 \text{ to } 9) \begin{bmatrix} N_{i,x} & 0 & 0 & 0 & 0 \\ 0 & N_{i,y} & 0 & 0 & 0 \\ N_{i,y} & N_{i,x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & N_{i,x} \\ 0 & 0 & 0 & -N_{i,y} & 0 \\ 0 & & & -N_{i,x} & N_{i,y} \\ 0 & 0 & 0 & & N_i \\ 0 & 0 & N_{i,x} & 0 & 0 \\ & 0 & N_{i,y} & -N_i & \end{bmatrix}$$

then U =

$$\frac{1}{2}\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\{\delta e\}^T [B]^T[D][B]\{\delta e\}\, dx\, dy$$

$$= \frac{1}{2}\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\{\delta e\}^T [K_e]\{\delta_e\}$$

in which $[K_e] = \int_{-a/2}^{a/2}\int_{-b/2}^{b/2}[B]^T[D][B]\,dx\,dy =$ element stiffness matrix. (63)

Since dx dy = |J| dξ dη , where |J| is the determinant of the Jacobian matrix, the element stiffness matrix can be expressed in local natural coordinates of the element. From equation (63),

$$[K_e] = \int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]\,|J|\,d\xi\, d\eta$$

III.3. Element Mass Matrix :

The potential energy of inertia forces for the element is obtained from equation (41)

$$V_i = -\iint_{Ae}\{u\}^T\{X\}dA.$$

where, $[N] = [\,[N_1]\ [N_2] \ldots\ldots\ldots [N_8]\ [N_9]\,]$

$$\text{where } [N_i] = \begin{bmatrix} N_i & & & & \\ 0 & N_i & & & \\ 0 & 0 & N_i & & \\ 0 & 0 & 0 & N_i & \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix} \qquad (i = 1 \text{ to } 9)$$

$$\text{in which, } [P] = \begin{bmatrix} p & & & & \\ 0 & p & & & \\ 0 & 0 & p & & \\ 0 & 0 & 0 & I & \\ 0 & 0 & 0 & 0 & I \end{bmatrix}$$

Therefore,

$$V_{ie} = -\omega_n^2 \int_{-a/2}^{a/2} \int_{-b/2}^{b/2} \{\delta_e\}^T [N]^T [P]\ [N]\ \{\delta_e\} dx\ dy$$

$$= -\omega_n^2 \{\delta_e\}^T [M_e] \{\delta_e\}$$

$$\text{Where } [M_e] = \int_{-1}^{1} \int_{-1}^{1} [N]^T [P]\ [N]\ |J|\, d\xi\ d\eta = \text{element mass matrix}$$

where, $[N] = [\,[N_1]\ [N_2] \ldots\ldots\ldots [N_8]\ [N_9]\,]$

The various element matrices [Ke] and [Me] are assembled to obtain their respective global matrices [K] and [M] by incorporating boundary conditions during assembly stage.

Solution Process :

The minimization of ∏ in the equation (42) leads the following equilibrium conditions for the free vibration of the laminated plates.

([K] – ω 2[M]) {δ} = 0

V CONCLUSION

In the present work, the finite element formulation is used to study effect of cutout on the free vibration of laminated composite plates. The formulation and program developed are general in nature and can handle cutout of any shape. The numerical results are presented and discussed in above. The broad conclusions that can be made from the present study are summarized as follows:

The fundamental natural frequency changes only marginally if a small cutout (eitherof the two cutout ratios being small) is made in the plate. However, for intermediate and large size cutouts, the fundamental natural frequency increases rapidly; the amount of increase depends on cutout ratios in two directions.

For square laminate with square and rectangular cutout, beyond a/s = 0.4, the fundamental natural frequency increases slightly with increasing a/s. The implication is that for moderate cutout sizes as there is no major change in the stiffness, if one of the cutout ratios is fixed and the other is increased then there is very little variation in the fundamental natural frequency. This trend is in contrast to that with a square cutout in which the fundamental frequency increases very rapidly with an increase in the cutout size.

VI FUTURE SCOPE OF WORK

The possible extensions to the present study are as presented below:

The present investigation can be extended to forced vibration analysis plates with cutout.

The present investigation can be extended to free vibration analysis of stiffened plates with cutout.

The present study can be extended to free vibration analysis of shells with cutout.

In the present study the analysis is based on first order shear deformation theory (FSDT). The analysis can be modified to incorporate higher order shear deformation theory (HSDT).

The present investigation can be extended to free vibration analysis of folded plates with cutout.

The present investigation can be extended to free vibration analysis of plates with delamination around cutout.

The present study can be extended to free vibration analysis of plates with arbitrary shaped openings.

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